BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

ME G641: THEORY OF ELASTICITY & PLASTICITY

FIRST SEMESTER 2022-23

Comprehensive Examination: **PART-A (CLOSED BOOK) (2PM to 5 PM)**

Max Marks: 15 Duration: 60 minutes December 22, 2022

1. The exam is divided into Part-A (Closed-book type) and Part-B (Open-book type).
2. Part-A (16 Marks) and Part-B (24 Marks) are to be answered in separate answer sheets.
3. You have 1 Hour to answer Part-A. Early and late submission of Part-A is allowed.

Q1. Investigate what problem is solved by given Airy's stress function when applied to the region included in $y = \pm a$, $x = 0$ and $x = l$. **Sketch the beam and show the loading wrt *x*-*y* axis. [6]**

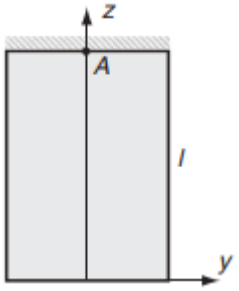
$$\phi = \frac{q}{4la^3}\left(\frac{xy^5}{5} - \frac{x^3y^3}{3} - \frac{2xy^3}{5}a^2 + x^3ya^2 + \frac{2x^3}{3}a^3 + \frac{a^4}{5}xy\right)$$

Q2. Consider the stretching of a prismatic bar *AB* under its own weight as shown below. **Find**

(a) the generated strains and

(b) the associated displacements (u_x, u_y, u_z) as a function of *x*, *y*, *z* coordinates based on the generalized Hooke's law.

Use gravity *g*, density ρ, area of bar *A*, Young's modulus as *E*, Poisons ration as 0.3. Note that the bar is hanging from the rigid support. **[6]**



Q3. A rectangular section steel beam, 50 mm wide by 20 mm deep, is used as a simply supported beam over a span of 2 m with the 20 mm dimension vertical. Determine (***a***) the value of the concentrated load which will produce initiation of yield at outer fibers of the beam. (***b***) If the central load is then increased by 20% find the depth to which yielding will take place at the centre of the beam span. (**Take** σ_y = 240 MPa) **[3]**

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FIRST SEMESTER 2022-23

Comprehensive Examination: **Part-B (OPEN BOOK)**

December 22, 2022 Max. Marks: 25 Duration: 2 Hours

Q1. In a proof test of a grinding wheel, the rotational speed is increased until the wheel bursts. The wheel is a disk of inner radius a = 100 mm, and an outer radius of b = 400 mm. The wheel is bonded to a steel shaft at the inner radius and has material properties ρ = 2000 kg/m^3, E = 12 GPa, υ = 0.32, and the ultimate strength σ_u = 21 MPa. Determine allowable rotational speed of the wheel (**rpm**) using a factor of safety 3. Assume that the steel shaft is rigid, and there are no forces acting on the outer periphery of the wheel. **[4]**

Q2. A steel tube 2.5 m long has the cross-section shown in **FigQ2**. The tube is transmitting a torque of 200 N-m. Determine the average shear stress (in **MPa**) in each wall and the angle of twist (in **degree**) of the tube. Take t_1 = 3 mm, t_2 = 4.5 mm, t_3 = 7.5 mm, a = 30 mm, b = 60 mm, c = 75 mm, E = 205 GPa and υ = 0.3 **[6]**

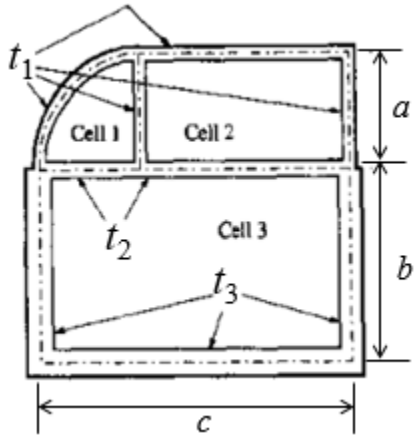

Fig Q2

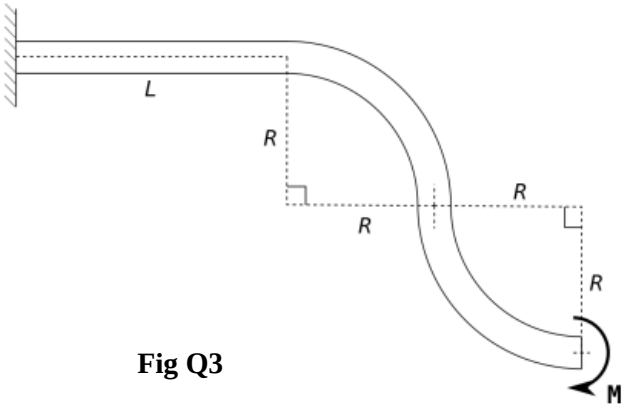

Fig Q3

Q3. Consider the planar beam shown in the **FigQ3**. The depth of the beam is much smaller than L, and R. Write bending moment equation separately for every segment and calculate the horizontal component of the deflection of the free end under the bending moment $\boldsymbol{M}$. The flexural rigidity of the beam is EI. **[6]**

Q4. Investigate what problem is solved by given Airy's stress function when applied to the region included in $y = \pm a$, $x = 0$ and $x = l$. Find the generated strains and displacement components (u_x, u_y) as a function of (x, y). Also obtain the equation of deflection curve and compute the maximum deflection. $\boldsymbol{I}$ is the moment of the inertia of beam cross-section. **[9]**

$$\phi = \frac{f}{24I}\left[(2a^3 - 3a^2y + y^3)\,x^2 + \frac{(2a^2 - y^2)y^3}{5}\right]$$