BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

ME G641: THEORY OF ELASTICITY & PLASTICITY

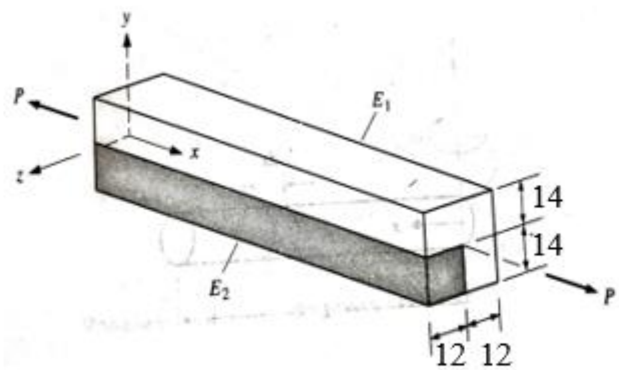
FIRST SEMESTER 2022-23

Mid Sem Examination (CLOSED BOOK)

Max Marks: 30 (30%) Duration: 90 minutes (11 AM to 12:30 PM) November 2, 2022

Q1. Derive the $\boldsymbol{\theta}$ – equilibrium equation in polar coordinates. Obtain the final form of equation and mention clearly what terms are neglected and why? **[3]**

Q2. A composite bar of rectangular cross section 24×28 mm, is loaded in tension by a force $P = 2$ kN as shown in Fig. The shaded part of the bar is made of a material where Young's modulus is E_2 = 210 GPa. The remaining part is made of a material with E_1 = 105 GPa. If the bar is to uniformly deflect in the x direction, determine the stresses in each material and the location of the loading axis relative to the center of the bar. **[5]**



Q3. For a beam (length l and rectangular cross-section with unit width and depth $2d$) under uniformly distributed loading (intensity of q per unit length), stress distribution is given by following equations

$$\sigma_x = \frac{q}{10 I_z}(5x^2 + 2d^2)y$$

$$\sigma_y = -\frac{q}{6 I_z}(2d^3 + 3d^2 y - y^3)$$

$$\tau_{xy} = \frac{q}{2 I_z} x(d^2 - y^2)$$

(a) Obtain the corresponding displacement field **(b)** Determine whether the obtained displacement field is compatible or not? If not, then determine the simplest correction that will cancel the unwanted term in stress functions. **[6]**

Q4. Given stress function (ϕ) is obtained from the superposition up to 4th order polynomial functions i.e. $(\phi_2 + \phi_3 + \phi_4)$ as trial Airy's function for a particular problem. Investigate what problem is solved by given function when applied to the region included in $y = 0$, $y = \pm d$, $x = 0$ and $x = l$.

$$\phi = \left(\frac{xy^3}{8d^2} + \frac{xy^2}{8d} - \frac{ly^3}{8d^2} - \frac{ly^2}{8d} - \frac{xy}{8} \right) q$$

Sketch the beam and show the loading wrt *x-y* axis. Highlight the important strong and weak boundary conditions in support of your conclusion. **No marks for writing redundant boundary conditions.** **[6]**

Q5. The displacement field is given by: $u_x = k(x^2 + 4z)$, $u_y = k(4x + 2y^2 + 2z)$, $u_z = 4kz^2$ where k is a constant small enough **(**$k = 10^{-3}$**)** to ensure applicability of the small deformation theory. **Determine the** **[10]**

(a) strain matrix at point ***P*** (3, 2, 4) **(b)** deviatoric strains associated **(c)** direction cosines associated with minimum principal strain **(d)** change in angle between two line segments **PQ** and **PR** at ***P*** having direction cosines before deformation as

PQ: $n_{x1} = 0.6, n_{y1} = 0, n_{z1} = 0.8$; **PR**: $n_{x2} = 0, n_{y2} = 1/\sqrt{2} = n_{z2}$