BITS Pilani K.K. Birla Goa Campus
Comprehensive Examination
Second Semester 2022-2023

ME G641- Theory of Elasticity and Plasticity

Date: 17/05/2023 | Total marks: 100
Time: 02:00 PM – 05:00 PM | No. of questions: 5

Note:

- *The exam is* ***closed book****. Students are not allowed to refer to any books/reference sheets during the exam.*
- *All variables, constants, and annotations carry the same meaning mentioned in the class/textbook.*
- *If necessary, make reasonable assumptions for solving the problems and state them clearly in the answer sheet.*
- *Use plane stress assumption for all problems unless explicitly specified*

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1. ($5 \times 3 = 15$ marks) Answer the following **with proper figures** and in not more than 5 sentences:
   (a) What is the difference between plane stress and plane strain problems in elasticity?
   (b) Show that the normal stress on an octahedral plane is the mean stress.
   (c) Explain the membrane analogy in torsion problems.
   (d) Explain the difference between Beltrami's yield criterion and von Mises yield criterion.
   (e) Explain three strain hardening rules.

2. (5 marks) State of stress for a point in 2D domain is given as $\sigma_x = a^2x, \sigma_y = b^2xy^2, \tau_{xy} = abxy$. Find the principal stresses.

3. (8 marks) The elasticity problem shown in Figure 1 can be solved using the stress function: $\phi = \frac{d_4}{6}xy^3 + \frac{b_3}{2}x^2y$. Find the constants d_4 and b_3 in terms of parameters s, c and l_0.

4. (12 marks) Through a shrink-fit process, a rigid solid cylinder of radius $r_1 + \delta$ is to be inserted into the hollow cylinder of inner radius r_1 and outer radius r_2 as shown in Figure 2. This process creates a radial displacement boundary condition $u(r_1) = \delta$. The outer surface of the hollow cylinder is to remain stress free. Determine the resulting stress field within the cylinder $(r_1 < r < r_2)$

5. (35 marks) (a) Identify the stress function $\phi = m \times h(x, y)$ that can solve the problem of bending of a beam with equilateral triangular cross-section as shown in Figure 3. Find the constant m in terms of P, α and G, assuming that the material is incompressible.

   b) Find all components of stresses.

6. (25 marks) (a) Identify the stress function $\phi = n \times q(x, y)$ that can solve the torsion problem of beam with hollow elliptical cross-section as shown in Figure 4. The outer and inner ellipses are concentric. The lengths of semi-major and semi-minor axes for outer and inner ellipses are a, b and ca, cb respectively, where c is less than 1. Find the constant n in terms of G, α and geometric parameters.

   b) Find the expression for torque T in terms of G, c, α and geometric parameters. (Hint: Parametric equations of an ellipse with semi-major/semi-minor axes a and b are $x = a\cos\theta, y = b\sin\theta$)

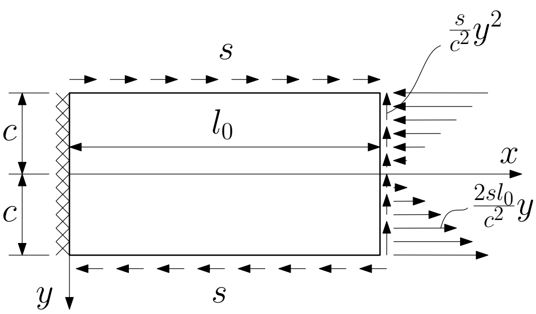

Figure 1: Domain for Problem 3

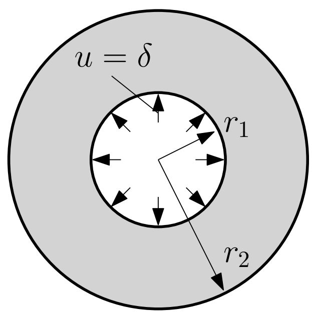

Figure 2: Domain for Problem 4

(P.T.O.)

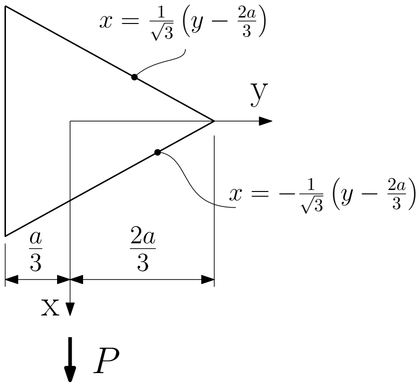

Figure 3: Cross-section for Problem 5

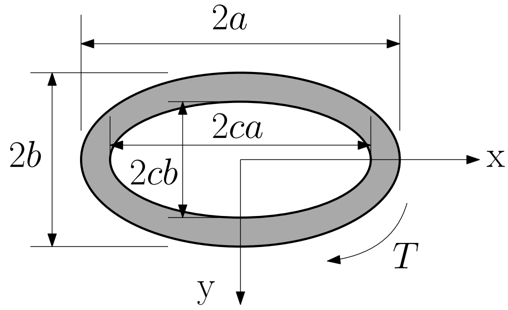

Figure 4: Cross-section for Problem 6

Given:

- Normal stress $\sigma_n = \sigma_x l^2 + \sigma_y m^2 + \sigma_z n^2 + 2\tau_{yz}mn + 2\tau_{xz}ln + 2\tau_{xy}lm$
- Airy stress functions in terms of stresses in 2-D rect. coordinates: $\sigma_x = \dfrac{\partial^2\phi}{\partial y^2}, \;\; \sigma_y = \dfrac{\partial^2\phi}{\partial x^2}, \;\; \tau_{xy} = -\dfrac{\partial^2\phi}{\partial x\partial y}$
- Expression for stresses in generic Axisymmetric problems are : $\sigma_r = \dfrac{A}{r^2} + B, \;\; \sigma_\theta = -\dfrac{A}{r^2} + B,$

  Strain-displacement relationships are: $\epsilon_r = \dfrac{\partial u}{\partial r}, \;\; \epsilon_\theta = \dfrac{u}{r} + \dfrac{\partial v}{r\partial\theta}, \;\; \gamma_{r\theta} = \dfrac{\partial u}{r\partial\theta} + \dfrac{\partial v}{\partial r} - \dfrac{v}{r}$

  and for plane stress conditions: $\epsilon_r = \dfrac{1}{E}\left(\sigma_r - \nu\sigma_\theta\right), \;\; \epsilon_\theta = \dfrac{1}{E}\left(\sigma_\theta - \nu\sigma_r\right), \;\; \gamma_{r\theta} = \dfrac{\tau_{r\theta}}{G}, \;\; G = \dfrac{E}{2(1+\nu)}$
- For cantilever beam-bending problems in 3D, the stresses can be chosen as:

$$\sigma_z = -E\left(L - z\right)\left(\kappa_x x + \kappa_y y\right), \quad \tau_{xz} = \frac{\partial\phi}{\partial y} + f(y) - \frac{1}{2}E\kappa_x x^2, \quad \tau_{yz} = -\frac{\partial\phi}{\partial x} - g(x) - \frac{1}{2}E\kappa_y y^2$$

  where $\kappa_x = \dfrac{I_x w_x + I_{xy} w_y}{E\left(I_x I_y - I_{xy}^2\right)}, \quad \kappa_y = \dfrac{I_{xy} w_x + I_y w_y}{E\left(I_x I_y - I_{xy}^2\right)}$

  The stress function ϕ should obey the compatibility relation

$$\nabla^2\phi = -\frac{dg}{dx} - \frac{df}{dy} + 2\nu G\kappa_x y - 2\nu G\kappa_y x - 2G\alpha$$

  and on the boundary, $\dfrac{d\phi}{ds} = \left\{\left[\dfrac{1}{2}E\kappa_x x^2 - f(y)\right]\dfrac{dy}{ds} - \left[\dfrac{1}{2}E\kappa_y y^2 + g(x)\right]\dfrac{dx}{ds}\right\}$
- For 3-D torsion problems:
  The stress function ϕ should obey the condition $\nabla^2\phi = -2G\alpha$

  Torque: $T = 2\displaystyle\int_A \phi dA + 2\sum_{i=1}^{n} k_i A_i$

  For ellipse: $I_x = \dfrac{\pi ab^3}{4}, \; I_y = \dfrac{\pi a^3 b}{4}, \; A = \pi ab$