BITS Pilani K.K. Birla Goa Campus
Mid-semester Examination
Second Semester 2022-2023

ME G641- Theory of Elasticity and Plasticity

Date: 14/03/2023
Time: 02:00 PM – 03:30 PM

Total marks: 50
No. of questions: 3

Note:

- *The exam is* ***closed book****. Students are not allowed to refer to any books/reference sheets during the exam.*
- *All variables, constants, and annotations carry the same meaning mentioned in the class/textbook.*
- *If necessary, make reasonable assumptions for solving the problems and state them clearly in the answer sheet.*
- *Use plane stress assumption for all problems unless explicitly specified*

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1. (20 marks) Figure 1 shows a cantilever beam subjected to uniform traction q applied at an angle α with respect to the $X-$axis. The elasticity problem can be solved using the stress function $\phi = \frac{a_2}{2}x^2 + b_2 xy + \frac{c_3}{2}xy^2$

   (a) Find the constants a_2, b_2 and c_3 in terms of q, c, l and α by substituting the boundary conditions on top and bottom surfaces.

   (b) Verify that the force and moment boundary conditions on the left and right surfaces are satisfied.

   (c) Assuming plane stress condition, find the displacements u and v in terms of the geometric, traction and material parameters (c, l, q, α, E and ν) for the boundary conditions $u = v = \frac{\partial v}{\partial x} = 0$ at $(x = l, y = 0)$.

2. (10 marks) A thin triangular plate carries uniformly varying load along its top edge as shown in Figure 2. The elasticity problem can be solved using the Airy's stress function

$$\phi = r^3 \left(a_1 \cos\theta + a_2 \sin\theta + a_3 \cos 3\theta + a_4 \sin 3\theta\right) \tag{1}$$

   Finding the constants a_{1-4} if $\alpha = \frac{\pi}{3}$.

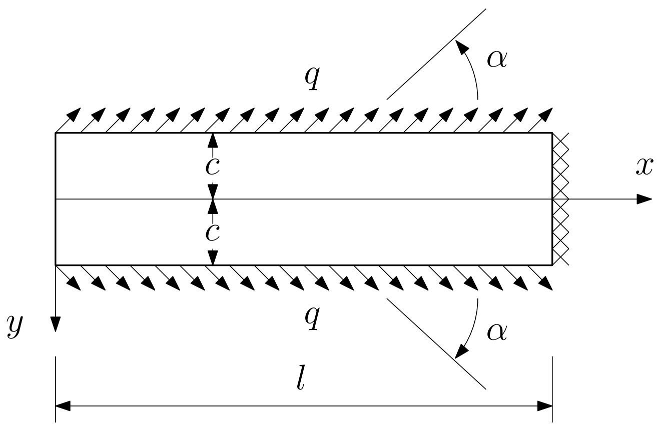

Figure 1: Domain for Problem 1

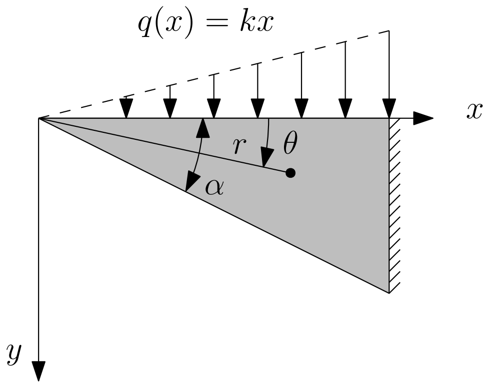

Figure 2: Domain for Problem 2

(P.T.O.)

3. (20 marks) Find the displacements u, v, w of the prismatic bar hanging due to its own weight (see Figure 3), for the boundary conditions $u = v = w = \dfrac{\partial u}{\partial z} = \dfrac{\partial v}{\partial z} = \dfrac{\partial v}{\partial x} = 0$ (Hint: $\sigma_z = \rho g z, \sigma_x = \sigma_y = \tau_{xy} = \tau_{yz} = \tau_{xz} = 0$)

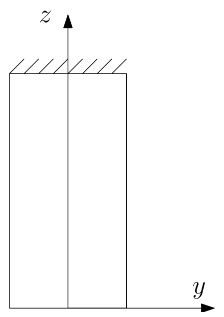

Figure 3: Domain for Problem 3

Given:

- In rectangular coordinates:
  2-D Airy stress functions:

$$\sigma_x = \frac{\partial^2 \phi}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 \phi}{\partial x^2}, \quad \tau_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y}$$

  Stress-strain relations:

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}, \quad \epsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_z}{E}, \quad \epsilon_z = \frac{\sigma_z}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E},$$
$$\gamma_{xy} = \frac{\tau_{xy}}{G}, \quad \gamma_{yz} = \frac{\tau_{yz}}{G}, \quad \gamma_{xz} = \frac{\tau_{xz}}{G}, \quad G = \frac{E}{2(1+\nu)}$$

  Strain-displacement relations:

$$\epsilon_x = \frac{\partial u}{\partial x}, \quad \epsilon_y = \frac{\partial v}{\partial y}, \quad \epsilon_z = \frac{\partial w}{\partial z},$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \quad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

- In polar co-ordinates:
  Stress functions:

$$\sigma_r = \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2}, \quad \sigma_\theta = \frac{\partial^2 \phi}{\partial r^2}, \quad \tau_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \phi}{\partial \theta}\right)$$