BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

ME G641: THEORY OF ELASTICITY & PLASTICITY

FIRST SEMESTER 2023-24

Mid Sem Examination (CLOSED BOOK)

Max Marks: 60 (30%) Duration: 90 minutes (11 AM to 12:30 PM) October 11, 2023

Q1. A solid circular structural steel (τ_y = 150 MPa and G = 77.5 GPa) torsion member has diameter D = 120 mm. The member is subjected to opposing torques of magnitude T, which cause an angle of twist of 10° over the length l =1.2 m. Then the torsion member unloads elastically. Determine

(a) the torque T and the radius of elastic-plastic boundary

(b) the residual stresses at the outer radius and at the elastic-plastic boundary, **sketch** the residual shear stress distribution

(c) the permanent set of angle of twist (in ***degree***). **[15M]**

Q2. Investigate what problem is solved by given Airy's stress function when applied to the region included in $y = \pm a$, $x = 0$ and $x = l$. **Sketch the beam and show the loading wrt *x-y* axis. *In what respect this solution is imperfect*.** **[15M]**

$$\phi = \left(\frac{x^3 y^3}{12a^3} - \frac{xy^5}{20a^3} + \frac{xy^3}{10a} - \frac{x^3 y}{4a} - \frac{x^3}{6} - \frac{xy}{20} a \right) \frac{p}{l}$$

Q3. The cantilever beam (rectangular cross-section with unit width and depth 2c) is subjected to the uniformly distributed load of intensity ω and the concentrated load P at the free end as shown in FigQ3. Consider superposition up to 5th order polynomial function as trial Airy's function for the problem. **(a)** Determine the suitable Airy's stress function for the stress field developed under the loading. **(b)** Define all the coefficients of Airy's stress function using key boundary conditions. **(c)** Derive the expressions for stresses at any point (x,y) as a function of loadings and beam dimensions i.e. (ω, P, c, I_z). **[20M]**

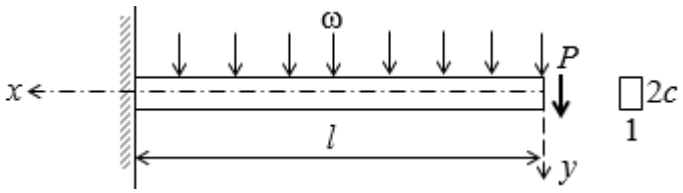
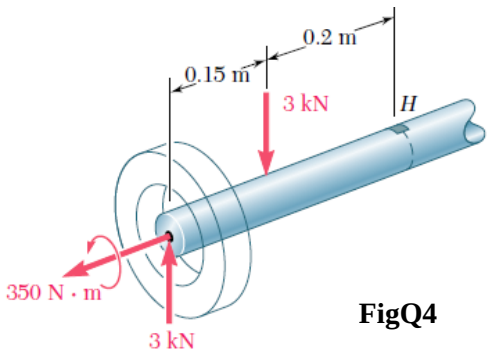


Fig Q3



FigQ4

Q4. The axle of an automobile is acted upon by the forces and couple as shown in **Fig Q4**. Knowing that the diameter of the solid axle is 32 mm and yield strength of the material is 350 MPa. Determine: **[10M]**

(a) the state of stress at point ***H*** located on top of the axle (on vertical diameter).

(b) the principal stresses at the same point.

(c) the factor of safety based on Von-Mises criteria.