

Birla Institute of Technology and Science - Pilani, KK Birla Goa Campus

First Semester, 2019-2020

Advanced Engineering Mathematics (MEG535)

Date: 30th September 2019, Time 9.00 AM: 10.30 AM, Total Marks: 50

1. Choose the correct answer

i) A and B are any matrices with the same number of rows. What can you say about the comparison of rank of A and the rank of the block matrix $[A \ B]$

- a) Rank of $[A] = \text{Rank of } [A \ B]$ b) Rank of $[A] \leq \text{Rank of } [A \ B]$
c) Rank of $[A] < \text{Rank of } [A \ B]$ d) Rank of $[A] \geq \text{Rank of } [A \ B]$

ii) Suppose $B = A^2$ in the above problem. How do those ranks compare?

- a) Rank of $[A] = \text{Rank of } [A \ B]$ b) Rank of $[A] \leq \text{Rank of } [A \ B]$
c) Rank of $[A] < \text{Rank of } [A \ B]$ d) Rank of $[A] \geq \text{Rank of } [A \ B]$

iii) If A is an $m \times n$ matrix of rank r, what is the dimension of the nullspace of $[A \ A]$

- a) $2m-r$ b) $2n-r$ c) $n-r$ d) $m-r$

(3 M)

2. Consider a 5×3 matrix A expressed in echelon form. The matrix is having 3 pivots.

i) Write down the dimensions of all the subspaces of the matrix A.

ii) Write down all the vectors in the nullspace of the matrix A.

(4 +1 M)

3. Calculate the row reduced echelon form of A, where

$$\begin{bmatrix} 1 & 2 & 2 & 3 \\ 1 & 2 & 4 & 3 \\ 1 & 2 & 6 & 3 \end{bmatrix}$$

Using your answer, describe the column space of A and nullspace of A.

(7 M)

4. Prove that non-zero eigenvalues of $A^T A$ and AA^T are the same. Also find a relationship between the eigenvectors of $A^T A$ and AA^T

(5 M)

5. Find the SVD for the matrix

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$$

(10 M)

6. Solve the following system of equations using “**method with least computational cost**” from the suitable methods. Specify the reasons for selecting the method. No marks will be given in the case of wrong selection of method.

$$2x_1 + 6x_2 - x_3 = 38$$

$$6x_1 + x_2 - 7x_3 = 34$$

$$x_1 - 7x_2 + 2x_3 = 20$$

(8+2 M)

7. Solve the following system of equations with Jacobi and Gauss-Seidel methods. Perform two iterations for each and comment on the convergence of these two methods. (**Exact solution $x_1 = x_2 = 2$**)

$$-3x_1 + 12x_2 = 18$$

$$10x_1 - 2x_2 = 16$$

(4+1M)

8. Find the inverse of the matrix A using Gauss-Jordan elimination method: $\mathbf{A} = \begin{bmatrix} 2 & 7 & 4 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ (5M)