Birla Institute of Technology & Science, Pilani
First Semester 2023-2024

Mid-Semester Test
(Regular)

Course No.	: MPBA G505
Course Title	: Statistics and Basic Econometrics
Nature of Exam	: Closed Book
Weightage	: 25%
Duration	: 1.5 Hours
Date of Exam	: 13/10/23

No. of Pages = 8
No. of Questions = 7

Note to Students:

- All parts of a question should be answered consecutively.
- Calculators are allowed.
- Show all the calculations and the final answer.
- Formula sheet and distribution tables are attached. Do not write anything on these sheets.

Q1. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Determine:

a. k

b. $P(X<3)$

c. $P(X>6)$

[3+2+2=7]

Q2. A survey of consumers showed that "How a company handles a crisis when at fault" is one of the factors influencing consumer buying decisions, with 73% claiming it is an influence. "Quality of product" also influences, with 96% of consumers stating that quality influences their buying decisions. "How a company handles complaints" was another factor, with 85% of consumers reporting it as an influence in their buying decisions. Suppose a random sample of 1,100 consumers is taken and each is asked which of these three factors influence their buying decisions. Assume the population is normally distributed.

a. What is the probability that more than 810 consumers claim that how a company handles a crisis when at fault is an influence in their buying decisions?

b. What is the probability that between 82% and 84% of consumers claim that how a company handles complaints is an influence in their buying decisions?

[3+5=8]

Q3. Previous experience shows the variance of a given process to be 14. Researchers are testing to determine whether this value has changed. Based on following data, test the null hypothesis about the variance. Assume the measurements are normally distributed. State business implications. Use $\alpha = .05$

52	44	51	58	48	49
38	49	50	42	55	51

[7]

Q4. Suppose a hypothesis states that the mean is exactly 50. If a random sample of 35 items is taken to test this hypothesis, what is the value of β if the population standard deviation is 7 and the alternative mean is 53? Also, what is the probability of committing type II error? Use $\alpha = .01$.

[7+1=8]

Q5. In manufacturing, does worker productivity drop on Friday? In an effort to determine whether it does, a company's personnel analyst randomly selects from a manufacturing plant five workers who make the same part. He measures their output on Wednesday and again on Friday and obtains the following results. The analyst uses and assumes the difference in productivity is normally distributed. Do the samples provide enough evidence to show that productivity drops on Friday? State business implications. Use $\alpha = 0.5$

Worker	Wednesday output	Friday output
1	71	53
2	56	47
3	75	52
4	68	55
5	74	58

[7+1=8]

Q6. Suppose 18 major computer companies operate in the United States and that 12 are located in California's Silicon Valley. If three computer companies are selected randomly from the entire list, what is the probability that one or more of the selected companies are located in the Silicon Valley?
[6]

Q7. Write the R code to create two vectors storing length of steel rods and test the hypothesis that mean lengths are equal. Assume variance is equal for the two population. Use $\alpha = 0.5$.
[6]

Formula sheet

Probability Functions

Binomial Distribution:

$$P(X) = \frac{n!}{X!(n-X)!} p^X \cdot q^{n-X}$$
$$\text{for} \quad 0 \le X \le n, \; \text{q} = 1 - \text{p}$$

Hypergeometric Distribution:

$$P(x) = \frac{({}_A C_x)({}_{N-A} C_{n-x})}{{}_N C_n}$$

Poisson Distribution:

$$P(X) = \frac{\lambda^X e^{-\lambda}}{X!} \quad \textbf{for} \quad X = 0,1,2,3,\ldots$$
where :
λ = *longrun average*
$e = 2.718282\ldots$ **(the base of natural logarithms)**

Exponential Distribution:

$$f(X) = \lambda e^{-\lambda x} \quad \textbf{for} \;\; X \ge 0, \; \lambda > 0$$
$$P(x \ge x_0) = e^{-\lambda x_0}$$

Bayes' Rule:

$$P(A_i|B) = \frac{P(A_i) \cdot P(B|A_i)}{P(B)} = \frac{P(A_i) \cdot P(B|A_i)}{P(A_1) \cdot P(B|A_1) + P(A_2) \cdot P(B|A_2) + \cdots + P(A_n) \cdot P(B|A_n)}$$

Z-test

Z value for sample mean

Z value for sample proportion

$$\boxed{Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}}$$

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p \cdot q}{n}}}$$

χ^2 formula for Single Variance

$$\boxed{s^2 = \frac{\Sigma(x - \bar{x})^2}{n-1}}$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

degrees of freedom $= n - 1$

Paired sample:

$$\bar{d} = \frac{\sum d}{n}$$

$$s_d = \sqrt{\frac{\Sigma(d - \bar{d})^2}{n-1}} = \sqrt{\frac{\sum d^2 - \frac{(\Sigma d)^2}{n}}{n-1}}$$

$$t = \frac{\bar{d} - D}{\frac{s_d}{\sqrt{n}}}$$

$$df = n - 1$$