

Birla Institute of Technology & Science (BITS), Pilani
2nd SEMESTER 2021-22,
Time Series Analysis and Forecasting MPBA G512
Comprehensive Examination (Closed Book)

Max. Time: 180 Minutes

Date: 12-05-2022

Max. Marks: 100

Q1. (a) The parameters of a GARCH (1, 1) model are estimated as $\omega = 0.000004$, $\alpha = 0.05$, and $\beta = 0.92$. What is the long-run average volatility and what is the equation describing the way that the variance rate reverts to its long-run average? If the current volatility is 20% per year, what is the expected volatility in 20 days? [12]

(b) A company uses the GARCH (1, 1) model for updating volatility. The three parameters are ω , α , and β . Describe the impact of making a small increase in each of the parameters while keeping the others fixed. [5]

Q2. Ten successive observations on a stationary time series are as follows:

45	47	49	51	48	54	57	58	54	55
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(a) Plot the observations and calculate the auto-covariance (c_1) and auto-correlation coefficients r_1, r_2, r_3 and r_4 for the given time series and plot a correlogram. Check whether the moving average process can be fitted to the data and which order of Moving average model will be suitable? [20]

(b) Calculate PACF(1), PACF(2) and PACF(3). Check whether the AR(1) process can be fitted to the data and obtain preliminary estimates of the parameters and also write down the equation of the AR(p) based on the value of the estimate(s)? [20]

Q3. Show that the autocorrelation function of the following MA(2) [8]

$X_t = \epsilon_t + 0.74\epsilon_{t-1} - 0.19\epsilon_{t-2}$, can be given by

$$\rho_k = \begin{cases} 0.38; & k = \pm 1 \\ -0.12; & k = \pm 2 \\ 0; & \text{Otherwise} \end{cases}$$

Q4. A random walk ($S_t, t=0, 1, 2, \dots$) starting at zero is obtained by cumulative sum of independently and identically distributed (i.i.d.) random variables. Check whether the series is stationary or non-stationary. [5]

Q5. For theoretically modeling the economic development of national economy scenarios the following 2 models for GDP increment are analyzed:

- a. $Y_t = Y_{t-1} + \epsilon_t$
- b. $Y_t = 8.5 Y_{t-1} - 15 Y_{t-2} + \epsilon_t$,

where Y - GDP increment

ϵ - White noise with zero mean and constant variance $\sigma^2 = 100$

t - Time (quarters starting with Q1, 1993)

Check by an algebraic criterion which one is stationary. [4+6=10]

Q6. Consider the process: $X_t - 0.4X_{t-1} - 0.45X_{t-2} = \epsilon_t + \epsilon_{t-1} + 0.25\epsilon_{t-2}$.

- a. Check whether the process is an ARMA (2, 2)? [6]
- b. Is the process stationary and invertible? [3]
- c. Calculate ρ_1 and ρ_2 . [3]

Q7. Write short notes on [2X4=8]

- a. Weak stationarity?
- b. What is the meaning of a unit root?
- c. Dickey-Fuller (DF) and augmented DF tests?
- d. What is a random walk (model)?