Birla Institute of Technology & Science (BITS), Pilani
2nd SEMESTER 2022-23,
Time Series Analysis and Forecasting MPBA G512
Comprehensive Examination (Closed Book)

Max. Time: 180 Minutes   Date: 08-05-2023   Max. Marks: 130

Q1. A series of 10 consecutive months' sales of a motorbike in a region are given as follows:

35, 34, 32, 29, 31, 32, 26, 21, 17, 12

a. Calculate the mean, autocovariances (c_1, c_2, c_3, c_4, c_5), autocorrelation coefficients (r_1, r_2, r_3, r_4, r_5) and partial autocorrelation coefficients (Φ_{11}, Φ_{22}, Φ_{33}). [30]

b. Based on the values of (r_1, r_2, r_3, r_4, r_5) and (Φ_{11}, Φ_{22}, Φ_{33}) which linear time series model should be used to fit on the given dataset? Also, identify the value of 'p (order of an AR process)' and 'q (order of a MA process)' and give the rationale for it. [10]

Q2. Discuss the theoretical patterns of ACF and PACF for different time series models. [6]

Q3. Consider an AR(1) process: $Y_t = \rho Y_{t-1} + \epsilon_t$; where $E(\epsilon_t) = 0$, $E(\epsilon_t^2) = \sigma_\epsilon^2$, and $E(\epsilon_t \epsilon_s) = 0$ for all $t \neq s$. Assume that Y_t is stationary. Derive a formula for $Cov(Y_t, Y_{t-s})$, the covariance of Y_t and Y_{t-s}, for s= 0,1,2,3, ... [10]

Q4. Let, u_t be white noise, where

$E(u_t) = 0 \ \forall t$

$E(u_t^2) = 20 \ \forall t$

$E(u_t u_{t-s}) = 0$ for all $t \neq s$

Let, $Y_t = u_t + 0.7u_{t-1} + 0.1u_{t-2}$. Determine the numerical values of

(a) $Var(Y_t)$ [4]

(b) The correlation between Y_t and Y_{t-1} [6]

(c) The covariance between Y_t and Y_{t-1} [4]

Q5. You want to fit an ARMA model to given data.

(a) Suppose you want to restrict attention to pure models (AR and MA) at first. How would you proceed for identifying lag orders? [4]

(b) If you allow for mixed structures of the type ARMA(p, q), you will probably use information criteria. Suppose the minimum AIC is found for ARMA(1,1) but the AR polynomial has its root less than one. What is your conclusion? [4]

(c) Now suppose the estimated model is perfectly stable. Which additional checks or tests would you apply to the model? [4]

Q6. a) Is the following process for y_t stationary?

$y_t = 3y_{t-1} - 2.75y_{t-2} + 0.75y_{t-3} + u_t$ [6]

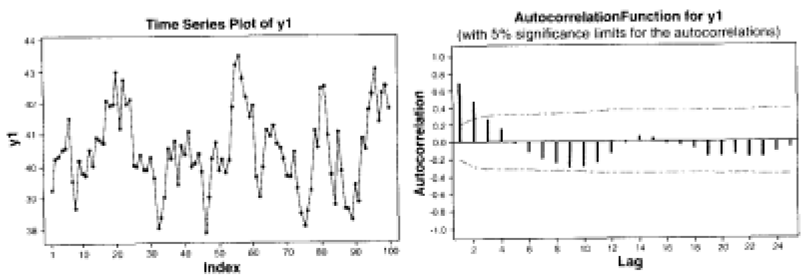
(b) You obtain the following estimates for an AR(2) model of some returns data
$y_t = 0.803y_{t-1} + 0.682y_{t-2} + u_t$; Where, u_t is a white noise error process.
By examining the characteristic equation, check the estimated model for stationarity. [6]

Q7. Describe the steps that Box and Jenkins (1976) suggested should be involved in constructing an ARMA model. [10]

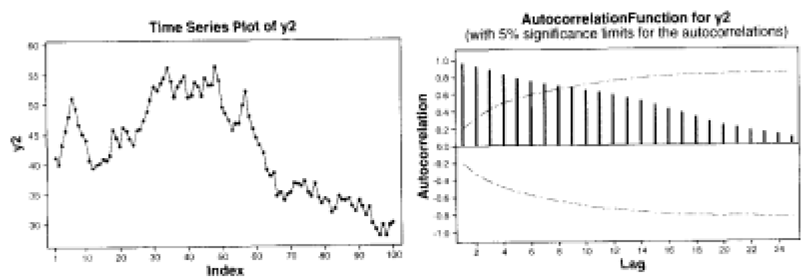
Q8. *A company uses the GARCH(1,1) model for updating volatility. The three parameters are ω, α, and β. Describe the impact of making a small increase in each of the parameters while keeping the others fixed.* [6]

Q9. *The parameters of a GARCH(1,1) model are estimated as $\omega = 0.00002$, , $\alpha = 0.08$, and $\beta = 0.87$. What is the long-run average volatility and what is the equation describing the way that the variance rate reverts to its long-run average? If the current volatility is 30% per year, what is the expected volatility in 20 days?* [12]

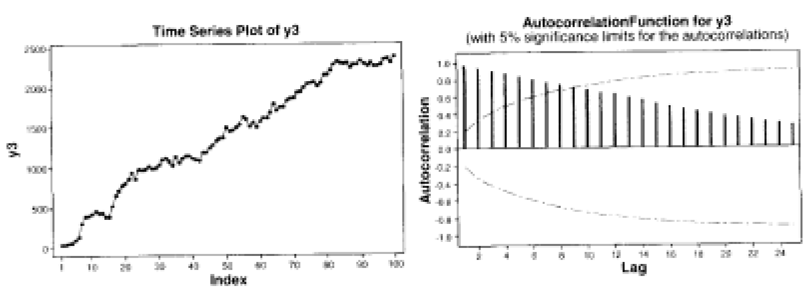
Q10. What is a stationary stochastic process? Explain the different types of stationary processes. Identify the stationary and nonstationary processes from the below figures and give the rationale. [8]



(a) $y_{1,t} = 10 + 0.75y_{1,t-1} + \varepsilon_t$



(b) $y_{2,t} = 2 + 0.95y_{2,t-1} + \varepsilon_t$



(c) $y_{3,t} = 20 + y_{3,t-1} + \varepsilon_t$