Birla Institute of Technology & Science (BITS), Pilani

1st SEMESTER 2022-23

PRESCRIPTIVE ANALYTICS WITH MATHEMATICAL PROGRAMMING MPBA G515

Mid Semester Examination – Part B (Open Book)

Max. Time: 70 Minutes | Date: 31-10-2022 (AN2) | Max. Marks: 40

1. Hero company produces bicycle frames using two fibreglass materials that improve the strength-to-weight ratio of the frames. The cost of standard grade material is Rs. 750 per yard and the cost of professional grade material is Rs. 900 per yard. The standard and professional grade materials contain two different amount of fibreglass, carbon fibre and Kevlar as given on the following table:

	Standard Grade	Professional Grade
Fibre Glass	84%	58%
Carbon Fibre	10%	30%
Kevlar	6%	12%

The company signed a contract with a bicycle manufacturer to produce a new frame with a carbon fibre content of at least 20 percent and Kevlar content of not more than 10 percent. To meet the required weight specification, a total of 30 yards of material must be used for each frame.

a) Formulate a Linear Programming Problem to determine the size of each grade of fibreglass materials that the company should use in each frame to minimize the total cost. Use graphical method. [8]

b) If the company is offered professional grade at Rs. 800/yard, will optimal solution change? What effect will the further lowering of price have on the optimal solution? [4]

2. A company is manufacturing two products, A and B. The manufacturing time required to make them, the profit, and capacity available at each work centre are as follows:

Product	Work Centre			Profit per unit (Rs.)
	Machine	Fabrication	Assembly	
A	1 hour	5 hours	3 hours	80
B	2 hours	4 hours	1 hour	100
Total Capacity	720 hours	1800 hours	900 hours	

If x_1 and x_2 represent the number of units of products A and B respectively, while S_1, S_2 and S_3 represent the slack variables indicating the unused capacity in the three work centres, we can state the problem as follows:

Maximise $\quad \text{Profit} = 80x_1 + 100x_2 + 0S_1 + 0S_2 + 0S_3$

Subject to

$$x_1 + 2x_2 + S_1 = 720$$
$$5x_1 + 4x_2 + S_2 = 1800$$
$$3x_1 + x_2 + S_3 = 900$$
$$\text{and } x_1, x_2, S_1, S_2, S_3 \geq 0$$

The simplex algorithm, applied to this problem, yields the following final tableau:

	Basis	x_1	x_2	S_1	S_2	S_3	b_i
x_2	100	0	1	5/6	−1/6	0	300
x_1	80	1	0	−2/3	1/3	0	120
S_3	0	0	0	7/6	−5/6	1	240
c_j		80	100	0	0	0	
Solution		120	300	0	0	240	
Δ_j		0	0	−30	−10	0	

Based on the above results answer the following questions:

a) Find the optimal values of all the decision variables and calculate the optimal value of the objective function of the primal problem.

b) Formulate the dual of this problem. [2]

c) Find the values of all variables of the objective function of the dual problem. [2]

d) Give the economic interpretation of the dual variables. [2]

e) If the cost of overtime in each of the departments is Rs. 15/hour. Would it be advisable to work either of the departments on an overtime basis? What would be the maximum amount of overtime authorised, if any? [2]

f) If the overtime costs were Rs. 8, how would the answer in (e) changes? [2]

g) Suppose change in price for the product A raise the profit for this product to Rs. 100/unit; would it change the optimal production plan? What is the maximum amount of change in profit for product A that would not cause a change in the optimum production plan? [2]

h) The company is planning to introduce a new product C with the following requirements per unit:

Machine: 2 hours

Fabrication: 3 hours

Assembly: 2 hours

What profit would be necessary before the company considers the production of this new product? [2]

3. A manufacturer of jeans is interested in developing an advertising campaign that will reach four different age groups. Advertising campaigns can be conducted through T.V., radio and magazines. The following table gives the estimated cost in paise per exposure for each age group according to the medium employed. In addition, maximum exposure levels possible in each of the media, namely T.V., radio, magazines are 40, 30 and 20 million respectively. Also, the minimum desired exposure within each age group, namely 13-18, 19-25, 26-35, 36 and older are 30, 25, 25 and 10 million. The objective is to minimize the cost of attaining the minimum exposure level in each group.

Media	Age Groups			
	13-18	19-25	25-35	36 and Older
T.V.	12	7	10	10
Radio	10	9	12	10
Magazine	14	12	9	12

Formulate the above as a transportation problem and find the optimal solution. [2+10=12]

~~[2+8=10]~~