Birla Institute of Technology & Science (BITS), Pilani

1st SEMESTER 2023-24

PRESCRIPTIVE ANALYTICS WITH MATHEMATICAL PROGRAMMING MPBA G515

Mid Semester Examination (Open Book)

Max. Time: 90 Minutes | Date: 09-10-2023 | Max. Marks: 60

Question1. Given below is a table obtained after a few iterations using the simplex method to solve a linear programming problem to maximize the total contribution margin from products A and B:

C_j / x_B	8.5	10.5	0	0	0	Solution
	x_1	x_2	S_1	S_2	S_3	
x_2	0	1	3/5	-2/5	0	300
x_1	1	0	-2/5	3/5	0	300
S_3	0	0	-1/5	-1/5	1	400

Provide short answers to the following questions, giving reasons as well:

i. Is the above solution optimal? [1]

ii. Is the above solution feasible? [1]

iii. Does the problem have an alternative optimal solution? If so, find another optimal solution. [2]

iv. Write the value of the objective function of the problem. [2]

v. What are the shadow prices for the three resources? [2]

vi. If S_1 represents the slack for the production capacity constraint, how much should the company be willing to pay for each additional unit of production capacity? [2]

vii. S_3 represents the slack for demand constraint. If the company is able to increase its total demand by 20 units, what will be the optimal mix and total contribution margin? [2]

Question2. A company produces three types of parts for automatic washing machines. It purchases castings of the parts from a local foundry and then finishes the parts on drilling, shaping and polishing machines. The selling prices of parts A, B and C are ₹ 8, ₹ 10, and ₹ 14. All parts made can be sold. Castings for parts A, B and C cost ₹ 5, ₹6, and ₹ 10. The company possesses only one of each type of machine. Costs per hour to run each of the three machines are ₹ 20 for drilling, ₹ 30 for shaping and ₹ 30 for polishing. The capacities (parts per hour) for each part on each machine are shown in the following table:

Machine	Capacity per hour		
	Part A	Part B	Part C
Drilling	25	40	25
Shaping	25	20	20
Polishing	40	30	40

The manager of the company wants to know how many parts of each type to produce per hour in order to maximize profits for the hour's run. Formulate the above as a linear programming problem (do not solve). [6]

Question3. The Sidon company produces four types of alloys which we label 1, 2, 3, and 4. Each type of alloy (per gram) requires three different types of metals as shown Table1.

Table1: Components of alloys

	Metal 1 (in grams)	Metal 2 (in grams)	Metal 3 (in grams)	Profit per gram
Alloy1	0.2	0.4	0.4	186
Alloy2	0.2	0.6	0.2	111
Alloy3	0.3	0.3	0.4	281
Alloy4	0.5	0.5	0	188

During the coming month, Sidon can acquire up to 2000 grams of metal 1, 3000 grams of metal 2, and 500 grams of metal 3. The unit costs are INR 5 per gram for metal 1, INR 5 per gram for metal 2, and INR 7 per gram for metal 3. The maximum demand for alloys 1, 2, 3, and 4 are 1000, 2000, 500, and 1000 grams respectively. The company wants to maximize its monthly profit by manufacturing the optimal quantity of alloys.

Let X_1 , X_2 , X_3 , and X_4 be the quantity of alloys (in grams) 1, 2, 3, and 4 to be manufactured. The corresponding LP formulation is given by

Maximize $186X_1 + 111X_2 + 281X_3 + 188X_4$

subject to constraints

$0.2X_1 + 0.2X_2 + 0.3X_3 + 0.5X_4 \leq 2000$ (metal 1 constraint)
$0.4X_1 + 0.6X_2 + 0.3X_3 + 0.5X_4 \leq 3000$ (metal 2 constraint)
$0.4X_1 + 0.2X_2 + 0.4X_3 \leq 500$ (metal 3 constraint)
$X_1 \leq 1000$ (alloy 1 demand)
$X_2 \leq 2000$ (alloy 2 demand)
$X_3 \leq 500$ (alloy 3 demand)
$X_4 \leq 1000$ (alloy 4 demand)
X_1 , X_2 , X_3 , and $X_4 \geq 0$

The Excel Solver output is provided in Table2

Table2: Sensitivity Report

Variable Cells						
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
A2	x_1	0	-36	186	36	1E+30
B2	x_2	1500	0	111	29.5	18
C2	x_3	500	0	281	1E+30	59
D2	x_4	1000	0	188	1E+30	188
Constraints						
Cell	Name	Final Value	Shadow Price	Constraint RHS	Allowable Increase	Allowable Decrease
A5	Metal 1	950	0	2000	1E+30	1050
A6	Metal 2	1550	0	3000	1E+30	1450
A7	Metal 3	500	555	500	100	300
A8	Alloy 1	0	0	1000	1E+30	1000
A9	Alloy 2	1500	0	2000	1E+30	500
A10	Alloy 3	500	59	500	750	250
A11	Alloy 4	1000	188	1000	2100	1000

Use primal–dual relationship to answer the following questions:

a. Write the optimal production plan (in terms of number of grams of alloys 1, 2, 3, and 4 manufactured). What is the objective function value? [2]

b. 200 additional grams of metal 3 can be imported and it would cost Sidon INR 20000. Should Sidon import this additional metal 3? State your answer clearly. [2]

c. What is the impact on the objective function value if the demand for alloy 3 is increased by 200 units? [2]

d. One of the customers of Sidon has placed order for alloy type 1. To maintain a long-term relationship Sidon would like to accept this order. How much should be the profit on alloy 1 so that Sidon can accept the order such that there will be no reduction from the current profit? [2]

e. One of the main competitors of Sidon goes bankrupt and using this opportunity, Sidon plans to increase the profits earned from different alloys to 279, 166.5, 421.5, and 282, respectively. State whether these changes to the profit will impact the current optimal production plan. [4]

f. Government imposes a restriction that Sidon can sell only 1000 grams of alloy 2. What will be the impact of this restriction on the optimal solution and the optimal profit? [2]

Question4. The data relating to production capacities of plants, orders from warehouses and freight costs for a company manufacturing consumer products are given below

Table3: Production capacities of plants

Plant	Unit Cost (₹)	Capacities (‘000 units)
X	12	20
Y	10	16
Z	15	25

Table4: Warehouse order sizes

Warehouse	Order Size (‘000 units)
P	15
Q	18
R	12
S	14

Table5: Freight costs/unit

	P	Q	R	S
X	3	1	2	4
Y	6	4	3	2
Z	4	5	7	3

From the above information, determine the optimum distribution for **minimising the total costs to make and ship** the product to warehouse. [16]

Question5: Solve the following assignment problem to find the **maximum total expected sales** [12]

Area / Salesman	I	II	III	IV
A	42	35	28	21
B	30	25	20	15
C	30	25	20	15
D	24	20	16	12