Birla Institute of Technology & Science (BITS), Pilani
1st SEMESTER 2023-24
FINANCIAL DERIVATIVES AND ANALYSIS MPBA G531
Comprehensive Examination (Closed Book)

Max. Time: 180 Minutes Date: 06-12-2023 Max. Marks: 40

Question1. Keeping all other parameters the same, if the dividend rate on the stock increases, which option depreciates less, the American call or the European call? Why? [2]

Question2. Explain the no-arbitrage and risk-neutral valuation approaches to valuing a European option using a one-step binomial tree. [2]

Question3. An option contract is a zero-sum game between the option buyer and the option writer. Explain this statement. [2]

Question4. The Black-Scholes model assumes constant volatility. How serious a shortcoming is this? [2]

Question5. Suppose that the parameters in a GARCH (1,1) model are $\alpha = 0.03$, $\beta = 0.95$ and $\omega = 0.000002$.

(a) What is the long-run average volatility? [1]
(b) If the current volatility is 1.5% per day, what is your estimate of the volatility in 20, 40, and 60 days? [1.5]

Question6. Assume that Asian Paints stock is currently selling for INR 1,750. There is a put option on Asian Paints with a maturity of 90 days and an exercise price of INR 1,800. The volatility of the stock price is 15%, and the risk-free rate is 9%. Form a risk-less hedge and calculate the price of a call option and a put option on the stock using Black-Scholes model. [3]

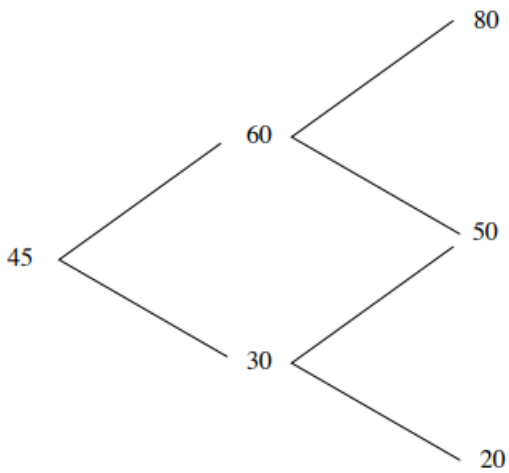
Question7. A stock price is currently \$40. Over each of the next two three-month periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 12% per annum with continuous compounding.

(a) What is the value of a six-month European put option with a strike price of \$42? [2]
(b) What is the value of a six-month American put option with a strike price of \$42? [1.5]

Question8. A fund manager has a well-diversified portfolio that mirrors the performance of the S&P 500 and is worth \$360 million. The value of the S&P 500 is 1,200, and the portfolio manager would like to buy insurance against a reduction of more than 5% in the value of the portfolio over the next six months. The risk-free interest rate is 6% per annum. The dividend yield on both the portfolio and the S&P 500 is 3%, and the volatility of the index is 30% per annum.

(a) If the fund manager buys traded European put options, how much would the insurance cost? [2]
(b) Explain carefully alternative strategies open to the fund manager involving traded European call options, and show that they lead to the same result. [3]
(c) If the fund manager decides to provide insurance by keeping part of the portfolio in risk-free securities, what should the initial position be? [1]

Question9. You are given the following tree of stock prices. In addition, the rate of interest per period is constant at 2%. Find the risk-neutral probabilities of the stock movements from each node on the tree. Are these probabilities the same? If not, explain whether the tree is a valid one [2]



Question10. How would you make a portfolio gamma-neutral? A delta-neutral portfolio has a gamma of 1,800 and a vega of −3,000. There exists an option with a delta of 0.3, gamma of 1.2, and vega of 2.8. How would you make this portfolio vega-neutral? [1+2 =3]

Question11. Consider an American call option on a stock. The stock price is $50, the time to maturity is 15 months, the risk-free rate of interest is 8% per annum, the exercise price is $55, and the volatility is 25%. Dividends of $1.50 are expected in 4 months and 10 months. Show that it can never be optimal to exercise the option on either of the two dividend dates. Calculate the price of the option. [3]

Question12. ABC Corporation can borrow at 6% fixed rate or at a floating rate of LIBOR + 50 basis points. GH Corporation can borrow at 8% fixed rate or at a floating rate of LIBOR + 100 basis points. Show that these two corporations can be better off by entering into an interest rate swap. Assume that the comparative advantage is equally shared by the two parties. [2]

Question13.

a) What is Itˆo's Lemma? [1]

b) Suppose that a stock price, S, follows geometric Brownian motion with expected return μ and volatility σ:

$$dS = \mu S\,dt + \sigma S\,dz$$

What is the process followed by the variable S^n? Show that S^n also follows geometric Brownian motion. [2]

Question14. Suppose that observations on a stock price (in dollars) at the end of each of 15 consecutive weeks are as follows: 30.2, 32.0, 31.1, 30.1, 30.2, 30.3, 30.6, 33.0, 32.9, 33.0, 33.5, 33.5, 33.7, 33.5, 33.2

Estimate the stock price volatility. What is the standard error of your estimate? [4]

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Standard normal probabilities (continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998