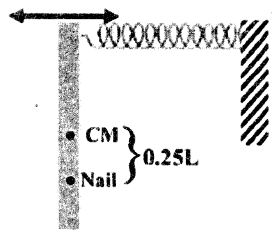
Birla institute of Technology and Science, Pilani, Pilani Campus
First Semester 2016-17
Comprehensive Examination
Mechanics, Oscillations & Waves

Course No.: **PHY F111** | Date: **02.12.16 AN**
Max. Marks: **60** | **PART - I (CLOSED BOOK)** | Time: **90 minutes**

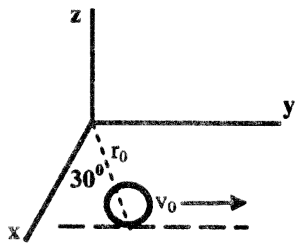
Instructions

- Write your ID number and name on the answer sheet.
- Do all the subparts of a question together. Answer a question on new page.
- Maximum time for this part is 90 minutes, however one can submit earlier and collect part-II (question paper and answer sheet).

Q.1. A uniform plank of length L and mass M is nailed at a distance $0.25L$ from its middle point 'CM' on a **frictionless horizontal plane** such that the plank can oscillate freely (as shown in figure by arrow) without friction about the nail. The one end of a **horizontal spring** of spring constant k is connected to the upper end of the plank as shown. The other end of the spring is fixed. Calculate the time period for small oscillation of the plank. **[10]**



Q.2. A hoop of radius a_0 and mass M rolls without slipping on the xy-plane with a constant velocity $\mathbf{V_0} = 0.5\hat{\jmath}$ m/s as shown. Consider its motion in rectangular coordinate system. At a certain time 't', the position of the contact point between the hoop and the plane is at a distance r_0 from the origin and makes an angle 30^0 with the x-axis as shown. Calculate the total angular momentum (only in vector form) of the hoop with respect to the origin in rectangular coordinate system, when $a_0 = 0.1$ m, $M = 0.5$ kg and $r_0 = 10$ m. **[10]**

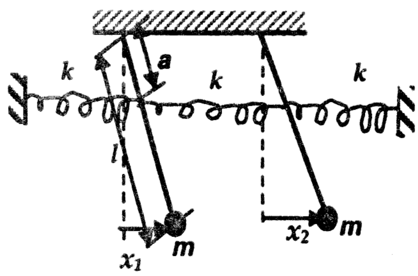


Q.3. Two identical pendulums consisting of equal masses m mounted on rigid, weightless rods of equal lengths l are coupled as shown in the figure. All three light springs (each of spring constant k) are unstretched when both rods are vertical and are attached to the rods at a distance 'a' from the pivot as shown in the figure. Let x_1 and x_2 be the linear displacements of the two masses from their equilibrium positions.

(a) Derive the differential equations of motion for x_1 and x_2 for small oscillations of the coupled system. (Neglect damping.)

(b) Find out the angular frequencies of the normal modes of the system.

[8+7]



Q.4. The displacement of a forced damped oscillator of mass m is given by $x(t) = A(\omega)\cos(\omega t - \delta)$, when it is subjected to a resistive force $-b\,v$ (where v is its velocity) and driven by a force $F(t) = F_0 \cos \omega t$.

(a) Derive the expression for the average of the power supplied by the driving force, in terms of ω, b & A.

(b) Find out the maximum value of the average power obtained in part (a) in terms of F_0 & b. **[5+2]**

Q.5. It is observed that a pulse requires 0.1 sec to travel from one end to the other on a long string. The tension in the string is provided by passing the string over a pulley to a weight whose mass is 100 times the mass of the string.

(a) What is the length of the string?

(b) What is the equation of the third normal mode? **[2+4]**

Q.6. Consider a damped system undergoing forced oscillations at an angular frequency ω.

(a) What is the instantaneous kinetic energy of the system?

(b) What is the instantaneous potential energy of the system?

(c) What is the ratio of the average kinetic energy to the average potential energy? Express the answer in terms of ratio ω/ω_0?

(d) For what values of ω, are the average kinetic energy and the average potential energy equal? What is the total energy of the system under these conditions?

(e) How does the total energy of the system vary with time for an arbitrary value of ω? For what value(s) of ω, is the total energy constant in time? **[1+1+5+1+4]**

BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI (RAJ)
FIRST SEMESTER 2016 - 2017

MECHANICS, OSCILLATIONS AND WAVES

COMPREHENSIVE EXAMINATION
Part - II (OPEN BOOK)

A

Course No: PHY F111 | **Max. Marks : 60**

Date: 02.12.16 | **Max. Time : 90 mins**

Instructions

- Check that question paper is printed on both sides. It consists of 24 questions.
- Write Id number and name on the answer sheet first.
- Write only the final answers in the boxes provided in the answer sheet. Only final answers will be checked.
- Always write answers with appropriate units. It carries half marks.
- Text books, lecture slides and own hand written note books are allowed.
- Rough work may be done in your own note books.
- Take acceleration due to gravity $g = 10\ m/s^2$, wherever required.

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1. A pendulum comprising mass m attached to a massless and inextensible sting of length l is hanging from the ceiling and performing SHM under gravity. Write down the equations of motion in polar coordinates. (2)

2. A particle of mass m is moving under the influence of a central force. Write the expression for time rate of change of force in polar coordinates in a vector form. (2)

3. A bicycle lies inverted on floor resting on its saddle and handle bar. A kid rotates the paddle with his hand such that the angular momentum of the bicycle about the axle of the paddle is L_0 . What is the angular momentum of the bicycle about the axle of its rear wheel which is distance R from the axle of the paddle. (2)

4. A flywheel of radius R and moment of inertia Mk^2 is mounted on a fixed axis. A massless rope is wound around the flywheel and to its free end mass m is tied which descends under gravity. What is the speed of m as it descends through height h. (2)

5. For a damped oscillator with damping coefficient γ and time period $10\ s$, the quantity $ln(A_0/A_n)$ is plotted against n, where A_n is the amplitude of the n^{th} oscillation. If the slope of the plot is 5, what is the damping coefficient γ? (2)

6. An elastic string of negligible mass, stretched so as to have tension $50\ N$, is attached to two fixed points, a distance $5\ m$ apart and carries 9 equally spaced particles of mass $5\ gm$ each. What is the angular frequency of the 10^{th} normal mode? (2)

7. Progressive waves of wavelength 4cm, traveling in a positive x direction are set up on a long string of mass density $0.1\ gm/cm$. The maximum amplitude at x = 2 cm is $\sqrt{2}$cm. If the displacement at $x = 2$cm is 1 cm at $t = 1$s, what is the tension in the string, assuming the phase δ to be zero? (2)

8. Given that a simple harmonic oscillator has speeds v_1 and v_2 when it is at locations x_1 and x_2 from the equilibrium position, write the expression for its (a) amplitude and (b) angular frequency in terms of given parameters (x_1, x_2, v_1, v_2). (2+2)

9. The total energy of a particle of mass m moving in one dimension is given by

$$E = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + A\left(1 - \cos(\alpha x)e^{-\beta x^2}\right)$$

Find the angular frequency of oscillation for small oscillation. (2)

10. An underdamped oscillator has time period of 2 seconds. The damping coefficient γ is such that the amplitude of oscillation goes down by 10% in one oscillation. What is the value of γ ? (2)

11. Two identical waves travelling in opposite directions are superposed

$$y_1(x,t) = 4.0\sin(3.0x - 2.0t) \qquad \& \qquad y_2(x,t) = 4.0\sin(3.0x + 2.0t)$$

Find the amplitude and angular frequency of the SHM that the element of the medium excutes at $x = 2.3$cm. (2)

12. The stationary wave in a string vibrating in the first harmonic is described by

$$y(x,t) = A\sin(2.5\pi x)\cos(5.0\pi t)$$

What is the speed of the wave through the string ? (2)

13. The suspension system of a car can be roughly modeled as a damped oscillator with a resonant frequency of $0.75Hz$. A road has bumpers every $20m$. At what speed shall the car experience violent bumps? (2)

14. Consider a critically damped oscillator with damping coefficient γ. The oscillator has initial position $x = 0$ and initial speed v_0. What is its furthest distance from the origin? (2)

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15. Consider a cannonball of mass m shot through the surface of Jupiter. The cannonball tears through the surface of Jupiter making a tunnel that runs parallel to the diameter at a distance b. Assume that the tunnel is just the right size offers no viscous or frictional resistance to its motion. To the surprise of the satellite crew, despite the fact that the tunnel is off-center, the cannonball performs SHM. What is the frequency of oscillations if the mass of Jupiter is M and its radius is R? (3)

16. Consider a forced damped oscillator being driven by a harmonic force of peak value 500 N and driving angular frequency significantly smaller than the natural angular frequency. If the spring constant is 500N/m and the Q value is unusually large, what is (a) the approximate amplitude of the oscillation and (b) phase difference between displacement and harmonic force. (3)

17. A damped oscillator of mass m, spring constant $k = 1\ N/m$ and $Q = 50$, is driven by a force $F_0\cos(\omega_0 t)$ where $F_0 = 0.1\ N$ and ω_0 is the natural angular frequency of the system. What is the displacement x at $t = T/8$ where T is the time period. (3)

18. A damped oscillator of mass $m = 10$ gm, spring constant $k = 1N/m$ and $Q = 50$, is driven by a force $F_0\cos(\omega_0 t)$ where $F_0 = 0.1$N and ω_0 is the natural angular frequency of the system. What is the average power input? (3)

19. A coupled pendulum experiment is performed on a different planet, by hanging two bobs A and B of equal mass m at the bottom of two almost massless rods of length 5m and coupled with a spring of unknown spring constant (k) at a distance of 2m from the pivot. First bob A is held fixed and bob B is displaced by a small angle after which both the bobs are released. Period of small oscillations of B turns out to be 2 sec and it was found that every 10 sec B comes to a stop before regaining its oscillatory motion. What is the acceleration due to gravity on this planet? (3)

20. Consider an empty cylindrical can of mass 100 gms floating vertically. If the height of the cylinder is 20 cms and the base diameter is 10 cm, find the mass of water that should be added for the cylinder to execute SHM with time period of 1 second (Assume density of water to be $1gm/cc$ and $g = 10\ ms^{-2}$). (3)

21. Consider a spring mass system oscillating with a natural angular frequency of $\omega_0 = 2\pi\ s^{-1}$. The system is assumed to have no damping. If the system is driven by a harmonic force $F = A\cos\omega t$, find the percentage change in amplitude of the oscillator when the angular frequency of the driving force is changed from $\pi\ s^{-1}$ to $1.5\pi\ s^{-1}$? (3)

22. One end of a horizontal string is attached to a vibrating blade, (vibrating with a constant frequency) and the other free end passes over a pulley. A sphere of mass 2.5 kg hangs on the free end of the string. The string is vibrating in its second harmonic. A container of water is raised under the sphere so that the sphere is completely submerged. In this configuration, the string vibrates in its fifth harmonic. What is the Buoyant force exerted by water on the sphere (assume $g = 10\ ms^{-2}$)? (3)

23. The effective acceleration due to gravity at some lattitude (θ measured such that equator is $\theta = 0$ and north pole is $\theta = \pi/2$) causes a freely hanging pendulum to make an angle ϕ with the line joining the point of suspension to the centre of the earth. Find ϕ. (3)

24. A bug of mass m runs around the rim of a circular disc of radius R. The bug is running in the clockwise direction with a speed v with respect to the ground and the disc is rotating with angular speed ω in the anti clockwise direction. The moment of inertia of the disc is I. If the bug suddenly stops moving, find the new angular speed. (3)

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COMPREHENSIVE EXAMINATION (OPEN BOOK)

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Answer sheet for Part-II

ID. ____________________ Name: ____________________ Marks Obtained

Q. No	Answer	Q. No	Answer
Q1		Q15	
Q2		Q16	a) b)
Q3		Q17	
Q4		Q18	
Q5		Q19	
Q6		Q20	
Q7		Q21	
Q8 a)		Q22	
Q8 b)		Q23	
Q9		Q24	
Q10			
Q11			
Q12			
Q13			
Q14			

Recheck request, if any: ____________________