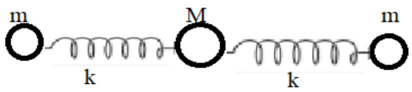
Birla Institute of Technology & Science -Pilani,
K K Birla Goa Campus
Semester1 2019-2020
Comprehensive Examination (Closed Book)
Mechanics Oscillations and Waves (PHY F111)

Date: 09/12/2019 **Duration: 9am-12 Noon**
Max Marks: 135

Answer all the questions. Writing with pencil is not allowed. **Make an index**, with question number and page number at the second page of the main Answer sheet.

1. Consider a point mass $\boldsymbol{M}$ connected to other two point masses, each having mass $\boldsymbol{m}$, via identical springs of spring constant $\boldsymbol{k}$. **[20]**



a. Write the equation of motion, by clearly defining the variables for each mass for the case of vibration of all the three molecules along the line joining them (Linear chain). [6]

b. Evaluate all the normal mode frequencies **[8]**

c. In each normal mode find the relative amplitudes (Note: For both **a, b** and **c** the final answer doesn't carry weightage unless the steps are correct). **[6]**

2. Consider two transverse waves where the vertical displacement (in cm) is given by $\boldsymbol{y_1 = Sin\,\pi(0.5x - 50t)}$ and $\boldsymbol{y_2 = Sin\,\pi(0.476x - 47.6t)}$. **[20]**

a. Distance between peak to peak of the modulated amplitude profile of the superposed wave is ………………. **[8]**

b. The number of waves contain in the modulated envelope of the superposed waves is …………………… **[6]**

c. Find the velocity by which the modulated profile moves. **[6]**

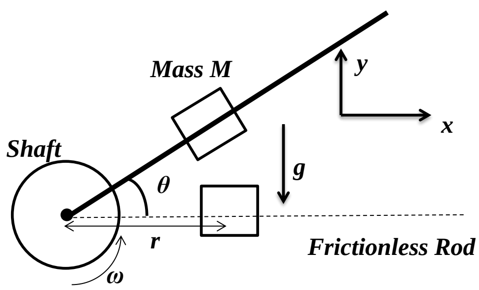
3. Consider a solid flat circular platform of radius R rotating with angular velocity $\omega = \omega\hat{k}$ in the anticlockwise direction (when viewed from above) with respect to an inertial reference frame. The platform is oriented in the XY plane such that the axis of rotation is in the z-direction. A solid block of mass M is sliding radially outwards in the platform with a constant velocity ($v = \dot{r}$=constant) with respect to the rotating reference frame. Coefficient of static friction on the platform is μ_s. Consider gravitational force in your calculations **[18]**

a. Identify the magnitude and direction of forces with respect to both inertial and non-inertial frame and tabulate them. (Write your answer in terms of, m, g, ω and v. Use the unit vectors $\hat{r}, \hat{\theta}$ *and* $\hat{k}$.**). [5]**

b. Diagrammatically show the trajectory of the solid block (*only for distance r<<R*) as it moves away from the origin with respect to both inertial and non-inertial observer. **[3]**

c. In the non-inertial frame, as the block is moving radially outwards, it tends to slip towards the $\hat{\theta}$ direction after reaching a critical radius r_{slip} (r_{slip}<R). Explain qualitatively why it will slip (**Hint:** maximum frictional force = $\mu_s N$) and write down the condition for slipping. Determine r_{slip} in terms of μ_s, g, v and ω. **[4+6=10]**

4. A frictionless rod on which block of mass M is free to slide is attached to a rotating shaft as shown in the figure. The rod is being rotated in the vertical plane ($X-Y$ plane) by the shaft in the counter clockwise direction with angular velocity ω which is changing slowly with time. Gravity acts vertically downwards along the -$\hat{y}$ direction. The rod is initially held in the horizontal orientation (θ=0) and the mass M located at a distance r from the centre of the shaft. **[27]**



a. Explain in brief the condition for preventing the mass M from sliding towards the centre $(r \rightarrow 0)$ as the shaft is rotated in the counterclockwise direction. **[4 marks]**

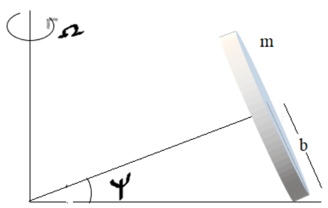
b. Show, with a neat sketch, all the forces acting on the block at any given instant in the inertial frame. Does the block have radial acceleration in the inertial frame? Explain**[3]**

c. By balancing the forces in the $\hat{r}$ *direction* in the rotating frame of the rod, determine the threshold angular velocity ($\omega_{threshold}$) for which the mass M remain stationary. **[10 marks]**

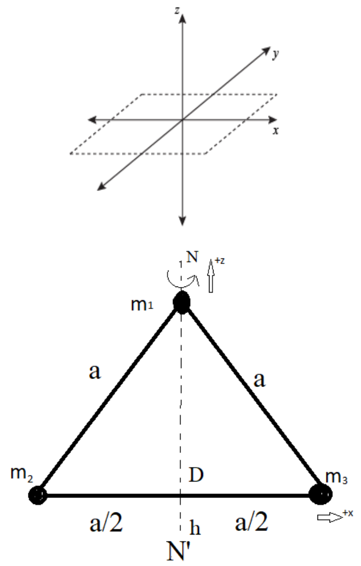
d. Calculate the radial acceleration, in the rotating frame, of the block for the following parameters $\{\omega = 2\omega_{threshold}$, mass of block = 1 Kg, g=10 m/s^2 and $\theta=30^0\}$ **[10 marks]**

5.

a. A massless axle (perpendicular to the surface of the flywheel disc) has one end attached to a fly wheel (a uniform disc of mass m and radius b), with the other end pivoted on the ground. The wheel rolls on the ground without slipping, with the axle inclined at an angle ψ with horizontal ground. The point of contact on the ground traces out a circle with frequency Ω (anticlockwise). Calculate the normal force between the ground and the wheel. **[15]**

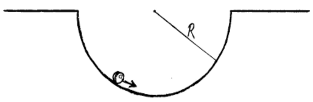


b. Three masses (m_1, m_2, m_3) are connected by three massless rods, in the shape of a rigid equilateral triangle of side a (shown in figure). The shape can rotate about the axis NN' (along z axis) with constant angular speed Ω. Calculate the inertia tensor and angular momentum of the three point masses about the point N' as origin. Sketch the direction of angular momentum at the point N'.($N'D = h$). **[10]**

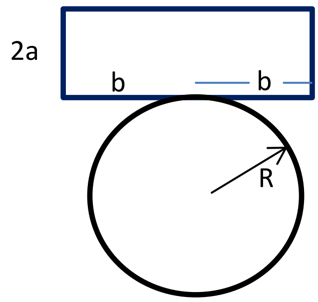


6.

a. A small ball with radius a and uniform density rolls without slipping near the bottom of a fixed cylinder of radius R. What is the frequency of small oscillation? Assume $a < R$. **[10]**



b. A rectangle of height 2a and width 2b rests on top of a fixed cylinder of radius R (see figure). The moment of inertia of the rectangle around its centre is I. The rectangle is given an infinitesimal tilt and then rolls on the cylinder without slipping. Under what conditions will it oscillate back and forth? Find the frequency of the small oscillation. **[15]**



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