BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
First Semester 2022-2023
COMPREHENSIVE EXAMINATION
Mechanics, Oscillations and Waves (PHY F111)

Date: 16.02.2023
Max. Marks: 45 **PART-A (CLOSED BOOK)** **Max. Time: 1 hr.**

Instructions:
- ✓ **Write your answer only in the attached answer sheet.**
- ✓ **Draw a star or double star on the TOP OF your ANSWER SHEET matching with the set of your question paper.**

Q.1 Draw the mean power absorption curve for the forced oscillator given by the equation, $\ddot{x} + 4\dot{x} + 36\,x = 20cos\omega t$, where the values of the parameters are in MKS system. The mass of the oscillator is 1 kg. Mark P_{max}, ω_0, and γ on the curve along with labeling the axis with suitable units. *No marks will be awarded if the value of parameters are not shown on the graph.* **[2+0.5+0.5]**

Q.2 When a violin string of frequency 210 Hz is struck, its amplitude is found to decay to 1/e of its initial value in 14 cycles. Which kind of damping is present? What is the value of γ? **[1+2]**

Q.3 A Barton's pendulum consisting of 11 pendula hangs from a common horizontal thread as shown in the figure below. The lengths of the 10 light pendula range from 0.1 m to 1.0 m increasing in the steps of 10 cm. The 11th pendulum which is a driving pendulum has a length of 0.4 m. It is oscillated horizontally with a maximum amplitude of 1 cm. What will be the amplitude of the pendulum which is having a phase lag of π/2 from the driving pendulum (γ = 2 N/m)? **[3]**

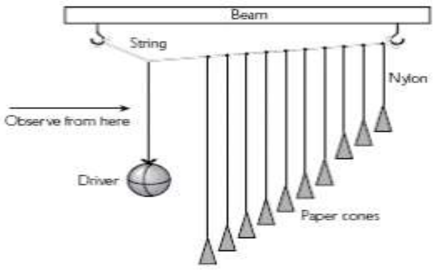

Fig. Q.3

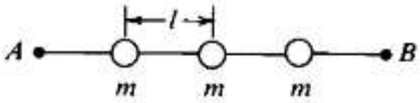

Fig. Q.4

Q.4 For a system of three coupled oscillators as shown in the figure, the lowest normal mode frequency is $(2-\sqrt{2})^{\frac{1}{2}}\,\omega_0$. Determine the amplitude ratios A_1: A_2: A_3 of the oscillators in this normal mode. **[3]**

Q.5 An extremely lightly damped oscillator (i.e. an undamped oscillator) consisting of a block of mass 0.5 kg and spring of spring constant k = 200 N m^{-1}, is at rest in its equilibrium position when a driving force of amplitude F_0 = 5 N and frequency 10 rad s^{-1}, is switched on. Write the exact transient solution for this oscillator. **[3]**

Q.6 Two identical simple pendulums are connected by a light coupling spring. Each pendulum has a length of 40 cm. When one pendulum is clamped, the period of the other is found to be 1.15 sec. What are the lower and higher normal modes of frequencies of the system? (Take g = 10 m s^{-2}) **[1+2]**

Q.7 An oscillator defined by the equation of motion, $\ddot{x} + 4\dot{x} + 4x = 0$, is released at 5 cm from the equilibrium with zero velocity. Write its solution. **[3]**

Q.8 A string of length L and total mass M is stretched to a tension T. Write the frequency of the lowest normal modes of oscillations. **[3]**

Q.9 If instead of a continuous string of length L and mass M as in **Q.8**, our system consisted of three individual oscillators of mass $M/3$ placed at a length $L/4$ from one another on a light, massless string under tension T, what will be the frequency of the lowest normal mode? **[3]**

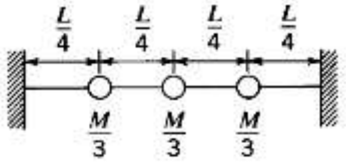

Fig.Q.9

Q.10 A uniform string of length 6.0 m and mass 0.01 kg is placed under a tension of 6 N. If the string is plucked transversely and is then touched at a point 0.3 m from one end, which frequencies persist? **[3]**

Q.11 A bowling ball is thrown down the alley with speed V_o. Initially, it slides without rolling, but due to friction, it begins to roll. What is its speed when it rolls without sliding? **[3]**

Q.12 A bead of mass m slides without friction on a rod that is made to rotate at a constant angular velocity ω. If the bead is moving such that $r = r_0 e^{\omega t}$, where r_0 is the initial distance of the bead from the pivot, find the power exerted by the agency that is turning the rod. **[3]**

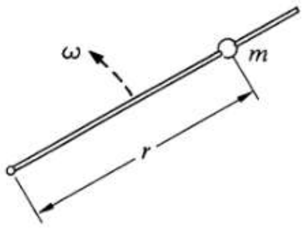

Fig.Q.12

Fig.Q.13

Q.13 A chain of mass 0.5 kg and length 50 cm is suspended vertically with its lowest end touching a scale. The chain is released and falls onto the scale. What is the reading of the scale when a length of chain 10 cm has fallen? Neglect the size of individual links. (Take g = 10 m s^{-2}) **[3]**

Q.14 A rocket of total mass 12,000 kg is fired vertically up with a speed of 960 m/s in the earth's gravitational field. The rocket ejects gas with a constant speed of 5000 m/s relative to itself. What is the velocity of the rocket after 10 seconds, when 8000 kg of fuel has been burnt out? Assume that g remains constant at 10 m/s^2. **[3]**

Q.15 Consider an isolated system of two masses m_1 and m_2 with position vectors given by $\vec{r_1}$ and $\vec{r_2}$. The masses are interacting under an attractive central force, $f(r)\hat{r}$ where $r = |\vec{r_2} - \vec{r_1}|$ and $f(r) > 0$. Write down the equation of motion for the reduced mass of the system. **[3]**

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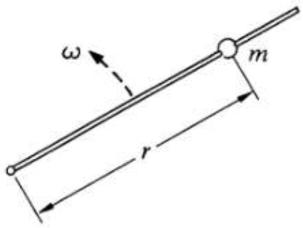

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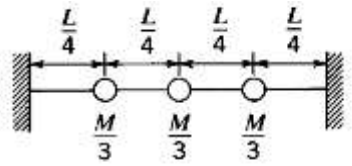

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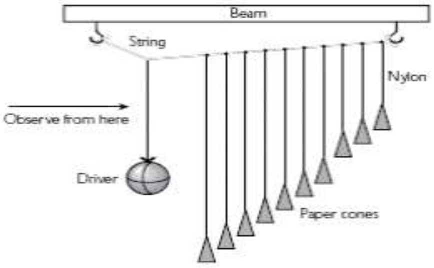
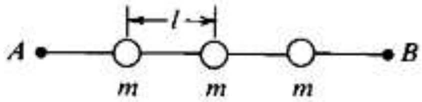


Fig. Q.10



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Max. Marks: 75 **PART- B (OPEN BOOK)** **Max. Time:2 hrs.**

Instructions:
- ✓ **Answer all parts of a particular question together.**
- ✓ **Write the final answer of each part in a box.**

Q.1 A pendulum consists of a bob of mass m suspended from a fixed frictionless pivot by a massless rigid rod of length l. The motion of the bob takes place in a vertical plane under the gravitational field g. Due to damping effect, the pendulum losses its energy at a rate (w.r.t. time) $\eta m\omega_0 l^2\dot{\theta}^2$, where ω_0 is the natural frequency of the pendulum, θ is the angular displacement and η is a positive numerical factor.

(a) Write down the expression for the total mechanical energy at any instant t, when the pendulum makes an angle θ.

(b) Obtain the equation of motion of the pendulum for small angle approximation in terms of η and ω_0.

(c) Now, the pendulum is set into forced vibrations by an external force $F(t) = F_0 \cos(\omega_0 t)$. Using $\theta(t) = A \sin(\omega_0 t)$ as a steady state solution to the equation of motion, derive an expression for the amplitude A in terms of η, m, ω_0 and F_0.

(d) The driving force is now removed and the pendulum is made critically damped. The bob is initially released from rest with an angle θ_0.
(i) Find the numerical value of η.
(ii) Find the maximum speed $V_{max.}$ in terms of l, θ_0 and ω_0. **[3 + 5 + 7+ (3+10)]**

Q.2 Two masses m and $3m$ are attached to two springs and to fixed points by three massless, identical springs of natural length a_0 and spring constant k along the x-axis, as shown in the figure below. At equilibrium, each spring has a length a. Consider small oscillations.

(a) Derive the equations of motion for transverse oscillations of each of the two masses, in terms of the given terms.
(b) Find out an expression of ω^2, where ω denotes the normal mode frequencies. **[6 + 9]**

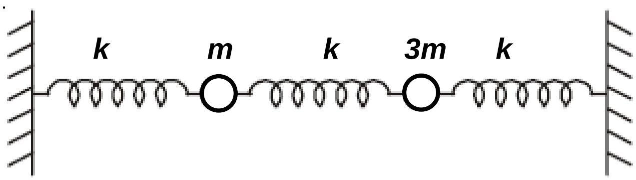

Fig. Q.2

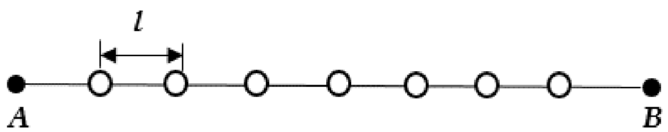

Fig.Q.3

Q.3 An elastic string of negligible mass, stretched so as to have a tension T, is attached to fixed points A and B a distance $8l$ apart, and carries seven equally spaced particles of mass m, as shown in the figure.
(a) The tension in the string is 9 N and each mass is of 100 g, separated to its next neighboring mass by a distance 10 cm. Find out the value of the angular frequency of the fourth normal mode of transverse oscillations. Additionally, write down the expression for the actual displacement of the fifth mass in this normal mode.
(b) Consider the fourth normal mode. Find out the ratio of the amplitudes of the transverse oscillations between the third- and first- masses.

(c) If we now consider that there are very large number of masses attached in a similar manner, what should be the maximum value of the angular frequency of the normal mode? **[(3 + 1) + 3 + 3]**

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Q.4 A man pushes a cylinder of mass m_1 with the help of a plank of mass m_2 as shown in the figure. There is no slipping at any contact. The horizontal component of the force applied by the man is F. The radius of the cylinder is R. Gravity (g) is downwards.

(a) Find the equations of motion for the cylinder and the plank assuming that their linear accelerations are a_1 and a_2 respectively. The angular acceleration is α.

(b) Find a_1, a_2 and α.

(c) Find the magnitude and directions of frictional forces at contact points. **[8 + 8 + 6]**

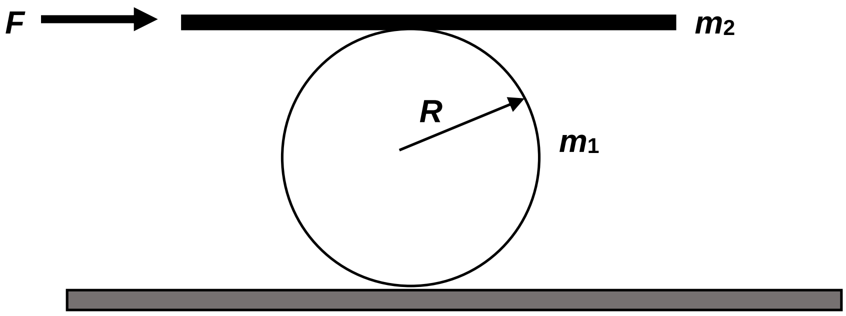

Fig. Q.4

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