BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

First Semester 2022-2023

MID-SEMESTER EXAMINATION

Mechanics, Oscillations and Waves (PHY F111)

Date: 06.01.2023

Max. Marks: 100 **(CLOSED BOOK)** **Max. Time: 90 Mins.**

Instructions:

✓ **Answer all parts of a particular question together.**

✓ **Write the final answer of each part in a box.**

Q.1 A particle of mass m moves in one dimension along the positive x-axis. It is acted on by a constant attractive force directed towards the origin with magnitude A and an inverse-square law repulsive force with magnitude B/x^2.

(a) Write down the total force acting on the particle in the vector form.

(b) Find the potential at x for attractive force only.

(c) Find the potential at x for repulsive force only.

(d) Find the equilibrium position of the particle (say, x_0).

(e) Expand the potential about $x = x_0$ up to 3rd term.

(f) Calculate the frequency of small oscillations (ω) about the equilibrium position of the particle? **[6×5 = 30]**

Q.2 Consider a spherical planet of radius R and mass M. Assume that the planet is non-rotating and has no atmosphere. A satellite of mass m is fired from the surface of the planet at 30° to the local vertical with speed v_0. In its subsequent orbit, the satellite reaches a maximum distance of $5R/2$ from the center of the planet.

(a) If v′ is the speed of the planet at distance $5R/2$, find v′ in terms of v_0.

(b) Find v_0 in terms of G (gravitational constant), M, and R. **[7+10]**

Q.3 Mass m is attached to a post of radius R by a string (see the figure Q.3). Initially, it is at a distance r_0 from the center of the post and is moving tangentially with speed v_0.

(a) The string passes through a hole in the center of the post at the top. The string is gradually shortened by drawing it through the hole.

(i) What can you say about the conservation of linear momentum, angular momentum, and energy. Give reasons.

(ii) What is the final speed of the mass as it hits the post?

(b) The string wraps around the outside of the post as the mass is whirled.

(i) What can you say about the conservation of linear momentum, angular momentum, and energy. Give reasons.

(ii) What is the final speed of the mass as it hits the post? **[7+2+7+2]**

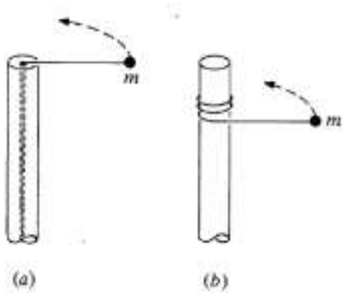

Fig. Q.3

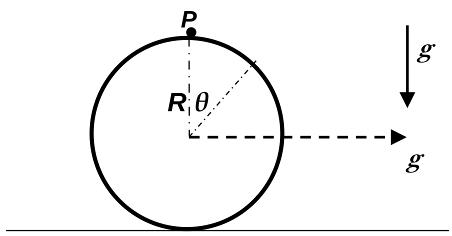

Fig. Q.4

Q.4 A point mass P (of mass M) is placed on the top of a smooth sphere of radius R which is placed on a smooth frictionless surface. The sphere is then pulled with a constant acceleration g in the horizontal direction through its origin (shown in the figure Q.4) and the point mass begins to slide down under the influence of the earth's gravity g which is vertically downwards.

(a) Write down the equations of motion of the point mass in polar coordinates.

(b) Calculate the numerical value of the angle θ that the point mass travels before its contact with the surface of the sphere is lost. **[7+8]**

P.T.O.

Q.5 A rocket is fired from a mobile launcher (at rest) with a ramp that is inclined at 30^0 above the horizontal, (see the figure Q.5). The initial mass of the rocket is M. The exhaust speed of the gas is u, which is constant and the fuel is also burnt at a constant rate of γ. Assume the earth to be flat, airless, and non-rotating with a constant acceleration due to gravity g that doesn't vary with height. Also, neglect the recoil of the mobile launcher by assuming the weight of the rocket is much less compared to the mobile rocket launcher.

(a) Write the equations of motion for the rocket in the horizontal and the vertical plane.

(b) Determine the rocket's velocity vector $\vec{V}(t)$ with respect to the initial firing position of the mobile rocket launcher before all its fuel is burned.

(c) Calculate the corresponding position vector $\vec{R}(t)$ from **(b)**. You may use the result $\int \log(a - \mathrm{bx})\,\mathrm{dx} = \left(x - \frac{a}{b}\right)\log(a - \mathrm{bx}) - x + C$ directly for the calculation of $\vec{R}(t)$. **[5+7+8]**

Fig. Q.5

** Best Wishes **