BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
FIRST SEMESTER 2023-24
PHY F111: *Mechanics Oscillations and Waves*
Midsemester Examination
Closed Book

Total marks: 100 | **Time: 90 mins** | ***Date: 12/10/2023***

Instructions:

- **All questions are compulsory**
- **Answer all parts of a particular question together**
- **Please box your final answer**
- **Write "The end" at the end of your last answer**

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1. A mass m is tied to a string and partially wraps around a frictionless pole of radius r. Someone is holding the other end of the string, keeping the string taut (see the figure). At $t = 0$, the mass m has speed v_0 in the tangential direction along the dotted circle of radius R. The person holding the other end of the string must ensure by moving the hand around the pole, that the string remains in contact with the pole subtending a constant angle at the center, and the mass continues to move on a circular path of radius R. Ignore gravity.

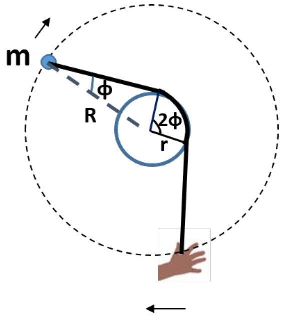

Figure 1: *Figure for Q1*

   (a) Write down the radial and tangential equation of motion for the mass m.

   (b) Find the angle Φ that the string makes with the instantaneous radially inward unit vector $\hat{r}$, in terms of tangential speed $v(t)$ and tangential acceleration $a(t)$.

   (c) What is the tangential speed $v(t)$ of the mass m?

   (d) Till what time will the velocity remain finite? **[5 + 5 + 7 + 3]**

2. A marble of radius b rolls back and forth in a shallow dish of radius R, where $R >> b$. Find the frequency of small oscillations. **[20]**

3. A person of mass m, is standing on a rail car A of mass M. The car moves with a velocity V. There is another identical rail car, B (of same mass M), moving with velocity V in front of A. The person jumps from the rail car A, with a velocity u with respect to it onto the car B. By assuming the conservation of momentum, find

   (a) The velocities of each of the rail cars A and B after the person lands on B

   (b) Now consider two people, each of mass m standing on A. They jump onto B keeping a relative velocity u with respect to A. Find (i) the velocities of A and B when both of them jump together (ii) The final velocity of A when they jump off separately. **[10 + (2 + 8)]**

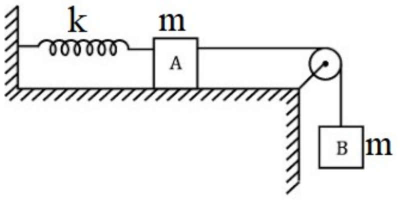

Figure 2: *Figure for Q4*

4. Consider a spring mass system as shown in figure. The coefficient of kinetic friction between the table and mass A is $\mu_k = 0.25$. The string is in-extensible and mass-less and the pulley is friction-less. The system is held with the spring at its relaxed length as shown in figure and then released.

   (a) When the mass B is released and it falls downwards what is the extension of the spring when the masses come to momentary rest for the first time?

   (b) What is minimum the value of the coefficient of static friction, μ_S, for which the masses will remain at rest once it has stopped for the first time?

   (c) If the string is cut when it has stopped for the first time, what is the maximal compression of the spring during the resulting motion? [**6 + 6 + 8**]

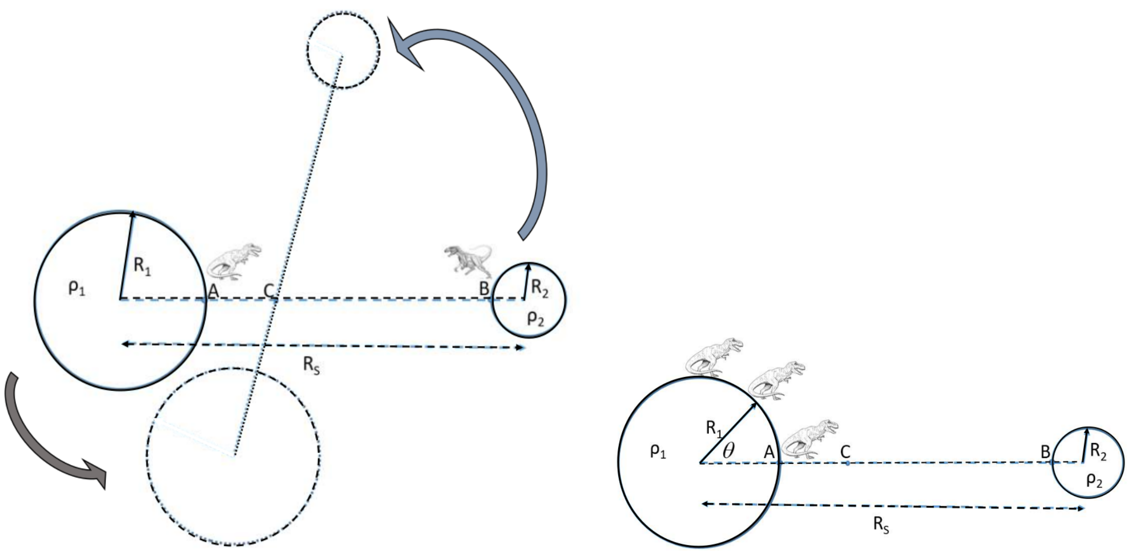

Figure 3: *Figure for Q5*

5. A binary planetary system is shown in the figure above. The densities and the radii of the planets are given as ρ_1, ρ_2, R_1, R_2 respectively. The point C is the center of mass position of the binary planetary system and R_s is the distance between the centers of the planets. R_s remains constant and the system rotates about the point C with a constant angular velocity.

   (a) Calculate the ratio of the tidal gravitational fields at point A and point B in terms of the given parameters.

   (b) What should be the relation between the densities of the planets so that the bigger planet may snatch a dinosaur from the point B ?

   (c) What should be the relation between the densities of the planets so that the smaller planet snatch a dinosaur located at A?

   (d) The tidal force is a conservative force and the corresponding expression for the potential energy of a dinosaur of mass m on the bigger planet is given by

$$V(r,\theta) = \frac{GMmR_1^2}{2R_s^3}(1 - 3\cos\theta)$$

   Here, M is the mass of the smaller planet that is responsible for the tidal force. Suppose the dinosaur on the bigger planet is initially at the north pole (see figure). If it slides down under the influence of only the tidal force, calculate the velocity of the dinosaur as it reaches the point A. (the surface of the bigger planet is assumed to be friction-less) [**5 + 5 + 5 + 5**]