BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
FIRST SEMESTER 2023-24
PHY F111: *Mechanics Oscillations and Waves*
Comprehensive Examination
Open Book

Total marks: 75 **Time: 120 mins** ***Date: 13/12/2023***

Instructions:

- **All questions are compulsory**
- **Answer all parts of a particular question together**
- **Please box your final answer**
- **Write "The end" at the end of your last answer**

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1. A particle of mass m moves on a plane. The mass executes a simple harmonic motion in the radial direction $\hat{r}$ with frequency ω_0 - the origin being the equilibrium position. Now if the plane is rotating about the vertical axis passing through the origin with angular veleocity Ω (direction of Ω is perpendicular upwards to the plane),

   (a) Write the force on the particle in the rotating frame of reference (in the vector form) [3]

   (b) Write the equation of motion in the radial (along $\hat{r}$) and tangential (along $\hat{\theta}$) directions along the plane. [4]

   (c) What is the conserved quantity that your obtain from the tangential component of the force equation? [4]

   (d) What is the most general solution of the radial equation if the tangential velocity is maintained to be $\dot{\theta} = 0$ of the particle for the cases (i) $\omega_0 > 2\Omega$ and $\omega_0 < 2\Omega$. [4]

2. Starting with the equation of motion for damped harmonic oscillator, with initial conditions: at $t = 0$, $x = x_0$ and $\dot{x} = v_0$

$$\ddot{x} + \gamma x + \omega_0^2 \dot{x} = 0, \quad \omega_0 = \sqrt{\frac{k}{m}}, \quad \gamma = \frac{b}{m}$$

   (a) Consider the underdamped case with a damped frequency ω_d. If ω_d differs from ω_0 by 20%, by what factor does the amplitude decay after two cycles? [3]

   (b) Consider the overdamped case and find the condition on the initial velocity v_o for which the overdamped mass never goes back to the origin. [4]

   (c) Consider a highly damped oscillator in a very viscous medium, where $\frac{2\omega_0}{\gamma} \ll 1$ (but non zero). What is the general solution for the motion at large enough times under this severely damped condition? Approximately sketch the total solution in this case. After what time will the particle's displacement fall by a factor $1/e$? [8]

   (d) For the highly overdamped case discussed above, for large enough times find the ratio of the damping force and the spring restoring force. [5]

3. (a) Consider a mass m executing undamped simple harmonic motion with natural frequency ω_o driven by an external force $F = F_o \cos \omega t$. At $t = 0$, the position and velocity of the mass are both zero. If the driving frequency ω is very close to ω_o then the solution maybe written as

$$x(t) = A \cdot t \cdot \sin(\omega_o t).$$

Find A in terms of known quantities? [6]

(b) A critically damped oscillator has mass 1 kg and the spring constant equal to 4 N/m. It is forced with a periodic forcing $F(t) = 2\cos 2t$ N.

i. Find the general solution when at $t = 0$, $x = 0$ and $\dot{x} = 0$. [6]

ii. Write the steady state solution for the oscillator. [2]

iii. Find the average power per cycle drawn from the forcing agent in steady state. [2]

iv. What is the full width at half maxima of the power resonance curve ? [2]

v. If the driving frequency is changed is there any value of the driving frequency when the amplitude of forced oscillator is a maximum (amplitude resonance) ? [2]

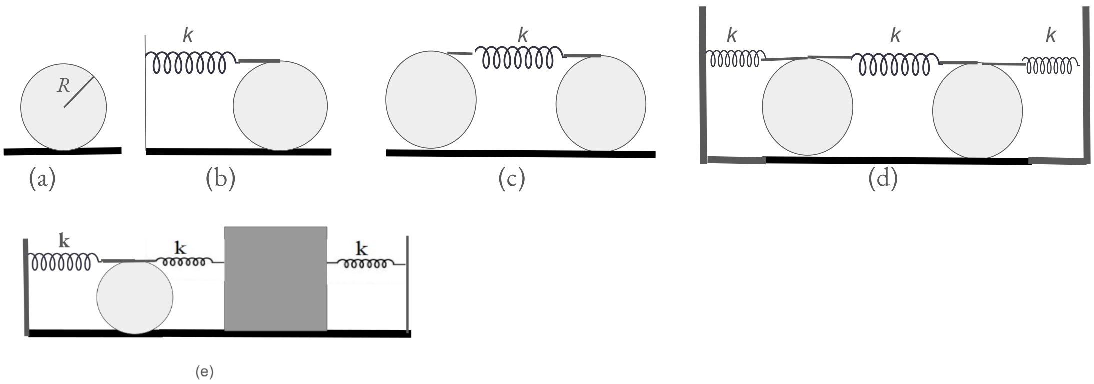

Figure 1: *Figure for Q4*

4. Consider a thin disc of mass M and radius R.

(a) What is the kinetic energy of the disc as it rolls without sliding [see figure (a)]? [1]

(b) The disc is attached to the wall by means of a massless spring with spring constant k as in figure (b). Show that for small rotations (without sliding) the motion is simple harmonic. What is the frequency of the simple harmonic motion? [3]

(c) Consider two such identical discs coupled by a spring of spring constant k as in figure(c). Treating this as a coupled system, write down the coupled equations of motion. Find the normal mode frequencies and interpret the values of the normal frequencies obtained. (assume here that rolling is without sliding). [6]

(d) Consider the arrangement as in figure (d) and write the coupled equations of motion. Hence, calculate the normal frequencies and the ratio of the amplitudes of oscillations of the two discs in each of the normal modes (assume rolling without sliding). [8]

(e) If the second disc is replaced by a cube of mass $M' = \frac{3}{8}M$, as in figure (e) assuming there is no friction between the cube surface and the floor while the disc is still rolls without sliding, what will happen to the normal mode frequencies ? [2]