BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

FIRST SEMESTER 2017-2018

Comprehensive Exam: Part-I (Closed Book:Quiz)

Course No: PHY F211 **Course Title: Classical Mechanics** **Date: 11.12.2017**

Max. Time: 45 mins. Total Marks: 30

Note: Write your final answer at the back side of the paper in the given box. Return it to the Invigilator when you complete it. No negative marking.
Questions 1 and 8 carry 6 marks. Questions 5 and 6 carry 1.5 marks. Rest questions carry 3 marks/each question.

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1. A bead slides on a wire in the shape of a cycloid (see the figure below) described by equations $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, $where$ $0 \leq \theta \leq 2\pi$. Write the Lagrangian of the system (take X-axis as the reference level for the Potential energy).

2. Write the Lagrange equation for the system in which dissipative forces are present (directly proportional to the velocity of the particle) in terms of Rayleigh's dissipation function $\mathcal{F}$.

3. A particle of mass *m* is subject to a central force $f = -kr$ with an angular momentum *l* , where symbols have their usual meanings. It has a stable circular orbit. To make its orbit parabolic, what is the energy required?

4. Here is a set of orthogonal transformations from $(x_1\ , x_2\ , x_3\ )$ to (x_1', x_2', x_3') coordinate axes;
$x_1' = x_1$, $x_2' = \frac{x_2}{2} + \frac{\sqrt{3}x_3}{2}$, $x_3' = \frac{-\sqrt{3}x_2}{2} + \frac{x_3}{2}$.

   (a) About which axis the rotation is taken?
   (b) What is the angle of rotation with respect to old coordinates?

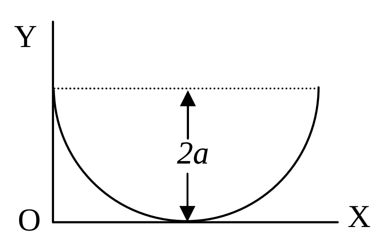
5. Which of the following statements is true regarding Coriolis force on earth, when a particle is thrown horizontally?
   A. It is maximum at the North Pole and minimum at South Pole.
   B. It is maximum at the North Pole and zero at equilateral.
   C. It is maximum at equilateral and zero at Pole.
   D. The sense of cyclone is clockwise in Northern hemisphere and anti-clockwise in Southern hemisphere.

6. Which of the following statements is false for Hamiltonian of a system?
   A. The Hamiltonian depends on the choice of generalized coordinates.
   B. The Hamiltonian is the total energy if the potential is velocity independent and the equations of transformation do not depend explicitly upon the time.
   C. The Hamiltonian is the total energy if the potential is velocity independent regardless whether the equations of transforms depend explicitly upon the time or not.
   D. If the Lagrangian is a constant in time, consequently, the Hamiltonian will also be constant in time .

7. Consider a canonical transformation $Q = (e^{-2q} - p^2)^{1/2}, P = cos^{-1}(pe^q)$. Find out $[P, P], [Q, Q]$ and $[Q, P]$.

8. A rigid body's orientation with respect to space axes is defined by Euler angles (45^o, 45^0, 45^0). What is the form of orthogonal matrix A which accomplishes these transformations.

9. A particle of mass *m* is moving in a central force field (potential *V(r)*). Find out the Hamiltonian *H* in the spherical coordinate system (r, θ, ϕ).



Answer Q1

Answer Q2

Answer Q3

Answer Q4

Answer Q5

Answer Q6

Answer Q7

Answer Q8

Answer Q9

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1. Which of the following statements is true regarding Coriolis force on earth, when a particle is thrown horizontally?
   A. It is maximum at equilateral and zero at Pole.
   B. It is maximum at the North Pole and minimum at South Pole.
   C. The sense of cyclone is clockwise in Northern hemisphere and anti-clockwise in Southern hemisphere.
   D. It is maximum at the North Pole and zero at equilateral.

2. Consider a canonical transformation $Q = (e^{-2q} - p^2)^{1/2}, P = cos^{-1}(pe^q)$. Find out $[P, P], [Q, Q]$ and $[Q, P]$.

3. A particle of mass *m* is subject to a central force $f = -kr$ with an angular momentum l , where symbols have their usual meanings. It has a stable circular orbit. To make its orbit parabolic, what is the energy required in terms of *m, k* and *l*?

4. A particle of mass *m* is moving in a central force field (potential *V(r)*). Find out the Hamiltonian *H* in the spherical coordinate system (r, θ, ϕ).

5. Here is a set of orthogonal transformations from (x_1, x_2, x_3) to (x_1', x_2', x_3') coordinate axes;
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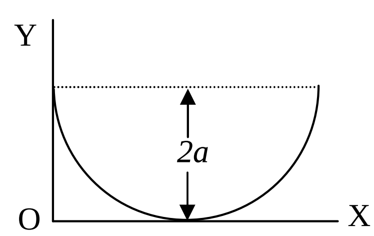


Answer Q1

Answer Q2

Answer Q3

Answer Q4

Answer Q5

Answer Q6

Answer Q7

Answer Q8

Answer Q9

BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

FIRST SEMESTER 2017-2018

Comprehensive Exam: Part-II (Closed Book)

Course No: PHY F211 **Course Title: Classical Mechanics** **Date: 11.12.2017**

Suggested Time: 75 mins. Total Marks: 45

Q1 A bead of mass *m* is sliding on a uniformly rotating straight wire about some fixed axis perpendicular to the wire in a force free space. Assuming the length of wire *1m* and the constant angular velocity $\omega = 2\,s^{-1}$. The constraint force which keeps the bead on the wire is found to be *mg* when the bead reaches to *0.25 m* from the axis of rotation. Find out the time when it reaches to this distance (*0.25 m*) assuming that the bead starts journey from the axis of rotation at *t=0*. Take $g=10\ m/s^2$. (10)

Q2 Consider an isotropic harmonic Oscillator subject to a central force, $f=-k\,r$, where symbols have their usual meanings, with a non-zero angular momentum $l \neq 0$.

(a) Draw the various curves of different potentials, V, V' and their sum, as a function of r with proper labelling.

(b) Mark the two turning points r_1 and r_2.

(c) Describe the stability of the system. (5+1+3)

Q3 Consider a transformation from one canonical variables (q,p) to other canonical variables (Q,P). The Hamiltonian in the former case is $H(q,p,t)$ and in the later case is $K(Q,P,t)$.

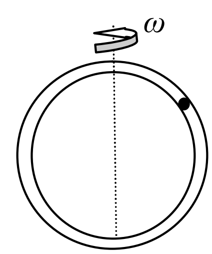
(a) Using Hamilton's principle, find out the equation which will be satisfied the transformation between the two sets of canonical coordinates (q,p) and (Q,P). From this equation, define the canonical transformation. (6)

(b) Using generating function $F = F_3(p, Q, t) + q_i p_i$, find out the equations of canonical transformation. (10)

Q4 A particle of mass *m* can slide without friction on the inside of a small tube which is bent in the form of a circle of radius *r*, as shown in the figure below. The tube rotates about a vertical diameter with a constant angular velocity, *ω*.

(a) Find out the Hamiltonian for the system.

(b) Find out the equation of motion in second order derivative from part (a). (4+6)



BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

FIRST SEMESTER 2017-2018

Comprehensive Exam: Part-III (Open Book)

Course No: PHY F211 **Course Title: Classical Mechanics** **Date: 11.12.2017**

Suggested Time: 60 mins. Total Marks: 45

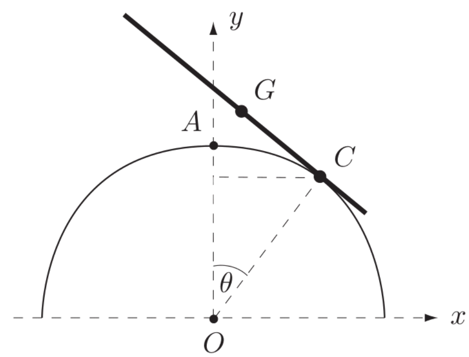
Q1 (i) A solid homogenous cylinder of radius, *r*, rolls without slipping on the inside of a stationary large cylinder of radius, *R*.

(a) Identify the generalized coordinate(s).
(b) Find the Lagrangian for the system.
(c) From (b), find the equations of motion.
(d) Find the period of small oscillation about the stable equilibrium position. (3+6+3+3)

Q2 Figure shows a uniform thin rigid plank of length *2b* which can roll without slipping on top of a rough circular log of radius *a*. The plank is initially in equilibrium, resting symmetrically on top of the log, when it is slightly disturbed.

(a) Find out the Lagrangian and Hamiltonian of the system.
(b) Find the period of small oscillations of the plank.

(10+5)



Q3 (a) The infinitesimal rotation associated with $\boldsymbol{\omega}$ of a rigid body can be considered as consisting of two successive infinitesimal rotations with $\omega_\phi = \dot{\phi}, \omega_\theta = \dot{\theta}$. Let us assume that the instantaneous orientation of the body specified by Euler's angle is $(30^o, 60^o, 0^o)$. Find out the components of $\boldsymbol{\omega}$ with respect to the body axes. (7)

(b) A particle is thrown up initially with initial velocity v_0 from earth at co-latitude θ, reaches a maximum height *h* and falls back to ground. Find out the deflection of the particle when it hits back to the ground. Assuming the angular velocity of earth is ω. Neglect the frictional effects of the atmosphere. (8)