BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

FIRST SEMESTER 2017-2018

MIDSEM Exam (Closed Book)

Course No: PHY F211 **Course Title: Classical Mechanics** **Date: 13.10.2017**

Max. Time: 90 mins. Total Marks: 90

Q1 A "Pedagogical Machine" is illustrated in the figure 1. All surfaces are frictionless. There is no external force in the horizontal direction.

(a) Write the constraint equation(s).
(b) Write the number of degrees of freedom and the generalized coordinates.
(c) Find the Lagrangian.
(d) Using Lagrangian undetermined multiplier(s), find out the acceleration of M_1. **(5+4+4+12)**

Q2. (a) Obtain the Lagrangian and equations of motion for the double pendulum illustrated in the figure 2, where the lengths of the pendula are l_1 and l_2 with corresponding masses m_1 and m_2. **(6+13)**

(b) If a cyclic coordinate q_j is such that dq_j corresponds to a rotation of the system of particles around some axis. Derive the corresponding conjugate momentum relation in terms of the torque and state the conservation principle. **(8)**

Q3. A particle moves without friction in a conservative field of force produced by various mass distributions. In each instant, the force generated by a volume element of the distribution is derived from a potential that is proportional to the mass of the volume element and is a function only of the scalar distance from the volume element. For the following fixed, homogeneous mass distribution, state the conserved quantities in the motion of the particle with proper justification:

(a) The mass is uniformly distributed in the plane $z=0$.
(b) The mass is uniformly distributed in the half plane $z=0$, $x>0$.
(c) The mass is uniformly distributed in a circular cylinder of finite length, with axis along the z-axis.
(d) The mass is uniformly distributed in a right cylinder of elliptical cross section and infinite length, with axis along the z-axis. **(2+2+3+3)**

Q4. (a) Starting from the Virial theorem $\bar{T} = -\frac{1}{2}\sum_i \overline{F_i . r_i}$, where symbols have their usual meanings, find the form of Virial theorem for a potential which obeys power-law ($V = a\, r^{n+1}$) **(5)**

(b) A particle of mass m is moving under the influence of inverse square force law.
Starting from $t = \frac{l^3}{m\,k^2}\int_{\theta_0}^{\theta} \frac{d\theta}{[1+e\cos(\theta-\theta')]^2}$, where symbols have their usual meanings, find out the transcendental equation (t as a function of θ) for the elliptical orbit. **(7)**

(c) At perigee of an elliptical orbit a particle experiences an impulse S in a direction which makes an angle 45^o from the radial direction, sending the particle into another elliptic orbit. Determine the new semi-major axis, eccentricity, and orientation of major axis in terms of the old. **(16)**

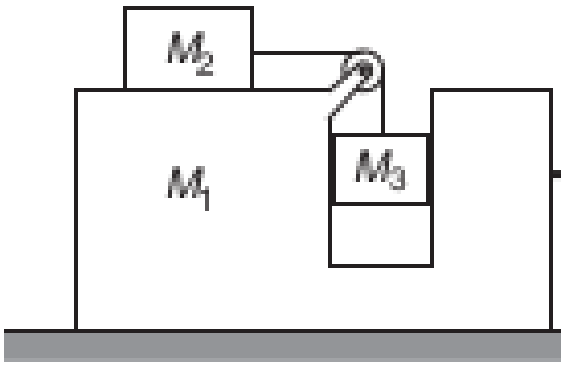

Figure 1

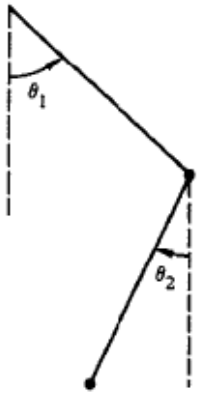

Figure 2