Birla Institute of Technology & Science – Pilani, K.K.B.G.C

First Semester 2019-2020

Classical Mechanics Mid-semester Examination

PHY F211 — September 30, 2019

Total Marks: 40 — Duration: 90 Minutes

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1. A simple harmonic oscillator of mass m and spring constant k is at rest at time $t = 0$. A force of the form $\alpha\frac{t}{T}$ acts on the particle for a time T. Find the energy of the oscillator at time T. **[6]**

2. A particle of mass m is constrained to move on a curve given by $y = x^2$ (with the gravitational force, mg, acting along the $-\hat{y}$).
   (a) Briefly describe d'Alembert's principle.
   (b) Use d'Alembert's principle to set up the equation of motion of the particle. (Use x coordinate of the particle as the independent coordinate.) **[2 + 5]**

3. A pendulum of mass m and length l is hung from a support of mass M which is free to move horizontally on a frictionless rail.
   (a) Write down the Lagrangian for the system after choosing appropriate generalized coordinates to describe the system. (Assume that the motion take place in a plane.)
   (b) What are the additive integrals of motion of the system?
   (c) For small oscillations, find the normal mode frequencies. **[4 + 4 + 6]**

4. The motion of a particle of mass m in the central force field, $\vec{F} = \frac{-k}{r^2}\hat{r}$, is given by $\frac{p}{r} = 1 + e\cos\theta$, where $p = \frac{L^2}{mk}$ and $e = \sqrt{1 + \frac{2EL^2}{mk^2}}$. Consider a flux of particles coming in from infinity with energy E (> 0).
   (a) Find the angular momentum L as a function of the impact parameter s.
   (b) Find the scattering angle as a function of s and E (Draw a neat diagram to support your calculation).
   (c) How does one experimentally measure the differential scattering cross section (DSCS)?
   (d) Find the DSCS for the present problem. **[2 + 5 + 2 + 4]**

Useful Formulae:

- $\frac{d\sigma}{d\Omega} = \frac{s}{\sin\Theta}\left|\frac{ds}{d\Theta}\right|$
- $\dot{x} + i\omega_0 x = e^{i\omega_0 t}\left(\int_0^t \frac{F(t')}{m} e^{-i\omega_0 t'} dt' + \dot{x}(0) + i\omega_0 x(0)\right)$