BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI (RAJASTHAN)

2022 - 2023 (SEMESTER I)
Classical Mechanics (PHY F211)
Mid-Semester Examination (Closed Book)
1st November 2022

Max. Marks : 90 **Duration : 1 hr 30 mins**

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1. Consider a *generalized* version of classical mechanics where the Lagrangian of a single particle system is of the form $L = L(q, \dot{q}, \ddot{q}, t)$. Assume Hamilton's least action principle holds for this generalized classical mechanics with the zero variation of both q and $\dot{q}$ at the end points.

   (a) Applying the methods of the calculus of variations, show that the corresponding Euler-Lagrange equation is of the form
   $$\frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{q}}\right) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) + \frac{\partial L}{\partial q} = 0.$$

   (b) Apply the above equation to a particle of mass m whose Lagrangian is given as
   $$L = -\frac{1}{2}q\left(m\ddot{q} + \frac{1}{2}kq\right),$$
   and derive the equation of motion.

   (c) The derived equation of motion is a very familiar one. Explain your result. **[15+10+10=35]**

2. A pendulum hung from the ceiling of a moving lift and its instantaneous position of the fulcrum being denoted by $z(t)$, then the Lagrangian of this system will be
   $$L(\theta, z, \dot{\theta}, \dot{z}) = \frac{1}{2}m\left(l^2\dot{\theta}^2 - 2l\dot{\theta}\dot{z}\sin\theta\right) + \frac{1}{2}m\dot{z}^2 + mgl\cos\theta + mgz.$$
   Construct the Hamiltonian of this system. **[15]**

3. Using Hamilton's principle, show that the shortest distance between two points on a plane is a straight line. **[10]**

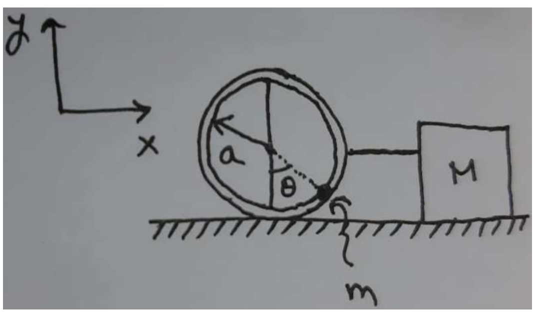
4. A particle is subjected to the potential $V(x) = -Fx$, where F is a constant. The particle travels from $x = 0$ at $t = t_i$ to $x = a$ at $t = t_f$. Assume the motion of the particle can be expressed in the form $x(t) = A + Bt + Ct^2$. Find the values of the constants A, B, and C. **[15]**

5. A block of mass M is rigidly connected to a massless circular track of radius a on a frictionless horizontal table as shown in the figure. A particle of mass m is confined to move without friction on the circular track which is vertical.

   (a) Construct the Lagrangian, using θ as one coordinate.

   (b) Determine the equation of motion from the Euler-Lagrange equations.

   (c) In the limit of small angles ($\sin\theta \simeq \theta, \cos\theta \simeq 1$), solve the equation of motion for θ as a function of time.



[5+5+5=15]