BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI (RAJASTHAN)
2017 - 2018 (SEMESTER I)
Electromagnetic Theory I (PHY F212)
(Comprehensive Examination: Closed Book)

Date : 8th December 2017 **A**

Max. Marks : 80 (Closed book 30) **Duration : 3 hrs**

Part I (Use only the last pages of the main answer copy for the rough work)

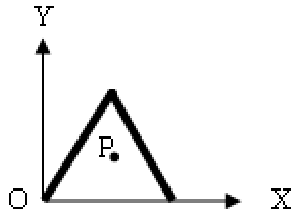
1a. Find the electric potential $V(r,\theta,\phi)$ outside an uniformly polarized solid sphere of radius R centered at the origin and having polarization $\vec{P}$ along the z-axis.

1b. The volume current density varies inside a wire (along z-axis) of radius a is given by $\vec{J} = J_0(1 - e^{-\gamma s})\hat{z}$, where J_0 and γ are constants and s is the radial distance from the axis of the wire. Calculate the total current passing through the wire.

1c. If a conducting metal sheet of surface charge density σ shows an electric field $\vec{E} = (3\sigma/\varepsilon_0)\hat{n}$ in one side of the metal surface what will be the electric field on the other side of the metal surface?

1d. Consider a long hollow solenoid of radius $R = 2\,\text{mm}$ and the number of turns per unit length $n = 5\text{cm}^{-1}$. If an alternating current of amplitude $5A$ and frequency $\nu = 10^4\text{Hz}$ is passed through the coils, calculate the induced electric field inside $(s < R)$ the solenoid.

1e. If a line element of linear charge density λ is forming the two sides of an equilateral triangle (side $= a$) in the $x - y$ plane (see figure) what will be the electric field at the centroid P of the triangle?



1f. If the electric field in a region is represented as $\vec{E} = Kr^3\hat{r}$, where $\vec{r}$ is the distance from the origin, find the charge density ρ.

1g. In the vicinity of a $p - n$ junction, the charge distribution is approximately given as $\rho(x) = \rho_0 \cosh(x/b)$, where b is a constant system parameter. Solve the Poisson's equation to calculate the electric field for the boundary condition that the electric field vanishes as $x \to \pm\infty$.

1h. A circular ring of radius $10\,\text{cm}$ is carrying a current $5A$, and small ring of radius $1\,\text{mm}$ is placed coaxially at a distance $20\,\text{cm}$. Calculate the approximate mutual inductance if two loops are in parallel.

1i. Find the value of the following integral along a parabolic path $x = y$; $x = z^2$:

$$\int_{(0,0,0)}^{(3,3,3)} \vec{\nabla}\left(x^2 + \frac{1}{9}xy + \frac{1}{81}xyz^3\right)\cdot \vec{dl}.$$

1j. The magnetic vector potential in a region is given by $\vec{A} = ay^2\hat{x}$. Find the volume current density in that region.

[3 × 10 = 30]

A

Answer sheet of Closed Book part

(Reminder: Use only the last pages of the main answer sheet for the rough work)

Name: | **ID:**

1a.

1b.

1c.

1d.

1e.

1f.

1g.

1h.

1i.

1j.

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2. A uniformly charged **solid** sphere of radius R carries a total charge Q, and is set spinning with angular velocity ω about the z-axis.

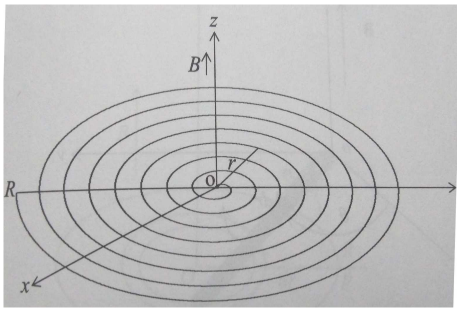
   (a) Calculate the magnetic dipole moment of the sphere?

   (b) Using the dipole approximation, calculate the vector potential $\vec{A}_{\text{dip}}(\vec{r})$ at large distances?

   (c) Calculate the vector potential **inside** the sphere. [**Hint:** You may directly use the results of Example 5.11 of your textbook (Griffiths' 4th Edition and 3rd Edition)]

[6+2+7 = 15]

3. (a) A spiral coil (lying on $x-y$ plane) of outer radius R and having total $N(\gg 1)$ number of homogeneously packed loops, is placed in a magnetic field $\vec{B} = 0.5\cos(100\pi t)\hat{z}$. Calculate the emf generated in the coil.



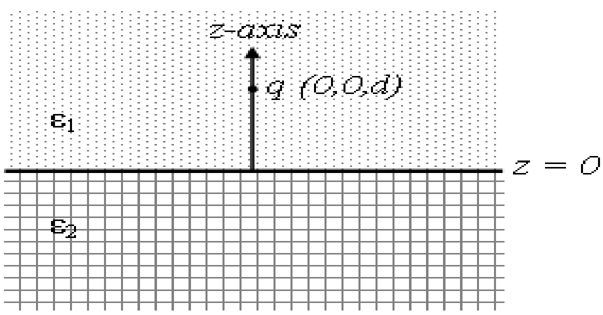
   (b) Two long cylindrical shells of radii 5mm and 10mm placed coaxially. If 5A current flows in both the shells along their length but in opposite directions, calculate the self-inductance of the system.

[5+5 = 10]

4. A point charge q is embedded within a semi-infinite linear dielectric (ε_1 for $z > 0$), a distance d away from a planer interface (at $x-y$ plane, $z = 0$) that separates the first dielectric medium from another semi-infinite linear dielectric (ε_2 for $z < 0$). Find the followings:

   (a) Surface charge density $\sigma(x, y)$ at the interface ($z = 0$) due to the polarization.

   (b) Potential $V(x,y,z)$ for both dielectric regions, as $V(z < 0)$ and $V(z > 0)$.



[9+6 = 15]

5. (a) A uniformly charged infinite plane slab of thickness $2d$ and volume charge density ρ is placed in $x-y$ plane such a way that $z=0$ represents the center of the slab. Find the electric field $\vec{E}$ at a point $P(z=a)$, for (i) $a<d$ and (ii) $a>d$.

(b) For $a>d$, another uniformly charged plane sheet is introduced parallelly at a distance $b(d<b<a)$ between the slab and point P such that the electric field $\vec{E}$ vanishes everywhere between the slab and sheet. Find the relation between the surface charge density σ of the sheet and ρ.

[6+4 = 10]

For your convenience:

$$\textit{Gradient}: \quad \nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$$

$$\textit{Divergence}: \quad \nabla\cdot\mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\textit{Curl}: \quad \nabla\times\mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

$$\textit{Laplacian}: \quad \nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$