

1. Evaluate the integral:

$$A = \int_V \left(\frac{1}{r-2} \right) \nabla^2 \left(\frac{1}{r^2} \right),$$

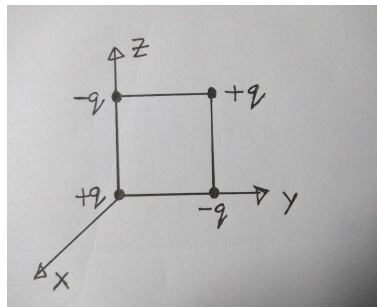
where V is the sphere of radius 2 centred at point $P(1, 1, 1)$.

[5]

2. (i) (a) Find the repulsive force between the 'northern' and the 'southern' hemispheres of a uniformly charged insulating solid sphere of radius R and total charge Q .
 (b) If the insulating sphere is replaced by a conducting one, what will be the repulsive force between the 'northern' and the 'southern' hemispheres?
 (ii) Find the capacitance of two concentric spherical shells of radii a and b ?

[(12+8)+5]

3. Four point charges are placed at the corners of a square of side a as shown in the figure.



(i) Explicitly derive the first three terms ($n = 0, 1, 2$) in the multipole expansion for the potential.

Given:
$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \left[\frac{1}{r} \int \rho(\vec{r}') d\tau' + \frac{1}{r^2} \int r' \cos \alpha \rho(\vec{r}') d\tau' + \frac{1}{r^3} \int (r')^2 \left(\frac{3}{2} \cos^2 \alpha - \frac{1}{2} \right) \rho(\vec{r}') d\tau' + \dots \right]$$

(ii) Two point charges q_1 and q_2 are placed at distances a and b , respectively, from the origin. Calculate the average potential over a spherical surface of radius R , centered at the origin (where $a < R < b$).

[12+8]

4. A point charge q is situated at a large distance r from a neutral atom of polarizability α . Find the force on the point charge due to the neutral atom.

[10]