BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
First Semester 2022-2023

MID-SEMESTER EXAMINATION
Electromagnetic Theory I (PHY F212)

Max. Time: 90 min. | **Total Marks: 90M** | **Date: 02.11.2022.** | **Closed book**

Instructions:

- ✓ ***The question paper consists of two types: objective type (no partial marking) & subjective type.***
- ✓ ***Answer all questions to the point and write all parts of a single question together.***
- ✓ ***Always box the final answer.***

Q1 Objective type questions **[15×2=30M]**

(a) What is the line integral of the gradient of $T = x^2+4xy+2yz^3$, along the parabolic path $z=x^2$, $y=x$, from the origin O $(0,0,0)$ to the point P $(1,1,1)$?

(b) Find the value of $\int_0^5 (x^2+3x+2)\delta(x+3)dx$

(c) Two conducting spheres of initial charges $4q$ and $-q$ are brought in contact and separated back again. If the radii of the spheres are $2r$ and r, what would the final charges on each spheres?

(d) The electric field inside a charge neutral sphere of radius R is $\mathbf{E} = kr^2\mathbf{r}$. What is the surface charge density if it is assumed to be uniform?

(e) Two infinite parallel planes carry equal surface charge densities of σ, separated by a distance d. Find the field between two planes and at a distance $2d$ left from the right plane.

(f) Write down the relation between bound (ρ_b) and free (ρ_f) volume charge densities for a homogeneous isotropic linear dielectric having electrical susceptibility χ_e?

(g) What are the bound charge densities for surface σ_b and volume ρ_b of a sphere of radius R carrying a polarization of $\mathbf{P}(\mathbf{r}) = \mathbf{r}/r^3$?

(h) In case of a charge distribution Q_i ($\mathbf{r}_i$), the total charge $\sum Q_i = 0$. If the dipole moment of this charge distribution with respect to a point A is $\mathbf{p}_a$, what will the dipole moment $\mathbf{p}_b$ with respect to another point B, which is separated by a distance $\mathbf{r}$ from *point A*?

(i) Two large metal plates each of area A are held at a small distance d apart. If each plate carries a charge Q, what is the electrostatic pressure on the plates?

(j) Two identical point charges $+q$ are located at (0,0,d) and (d,0,d), respectively. If an infinite grounded conducting plane is placed in the XY plane now, what is the force experienced by the charge q (0,0,d)?

Q2. (a) Find the repulsive force $\mathbf{F}_i$ between the '*northern*' and the '*southern*' hemispheres of a uniformly charged (insulating) solid sphere of radius R carrying total charge Q.

(b) If the sphere is replaced by a conductor, what will be the new force $\mathbf{F}_c$ in terms of $\mathbf{F}_i$? **[12+8=20M]**

Q3. (a) A point charge q is situated at a large distance $\mathbf{r}$ from a neutral atom of electrical polarizability α. Find the force on the point charge q due to the neutral atom.

(b) Two infinitely grounded metal plates make an angle $60°$ with each other, meet along the X-axis. One of the plate is in the XY plane and a point charge $+q$ is placed on the YZ plane at equidistant d from both plates. Schematically represent the image charges (position, magnitude and polarity) which can eventually replace the effect of surface charge distribution on the grounded plates for the field space.

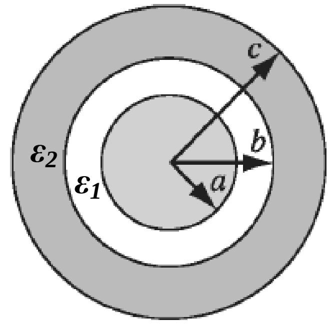
(c) Now, find the potential V (x = 0) on the YZ plane at distances $2d$ from both the plates. **[6+8+6=20M]**

Q4. A spherical conductor of radius a and uniform surface charge density σ is surrounded by two concentric thick spherical shells of liner dielectric materials of permittivity ε_1 and ε_2, respectively.

(a) Find the potential at the center with respect to the infinity.

(b) Find the surface charge densities σ at the interface ($r = b$) of two dielectrics.

(c) What is the total energy of this configuration? **[7+6+7=20M]**



In case you may need

$$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$