

**Electromagnetic Theory I (PHY F212): Midsem - PART B**

Date : 9 Oct 2023

Closed Book

Marks: 60

Duration max: 90 min

**All the parts should be answered together. All bold face characters represents vectors.**

1. (a) Define the Strokes theorem for a vector field  $\mathbf{v}$ .  
 (b) Test the Strokes' theorem for the vector field  $\mathbf{v} = (xy)\hat{\mathbf{x}} + (2yz)\hat{\mathbf{y}} + (3zx)\hat{\mathbf{z}}$  using the a triangular shaded area shown in the Figure 1. **[2+11 = 13]**

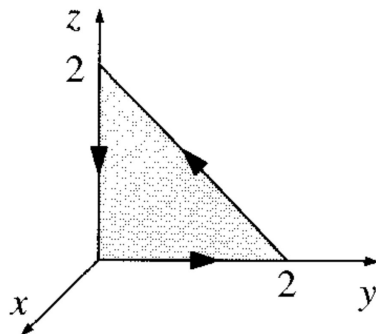


Figure 1:

2. A uniformly charged solid sphere of radius  $R$  and having a volume charge density  $\rho$  is surrounded by a thick concentric metal shell of inner radius  $a$  and outer radius  $b$ . The shell carries no net charge.
- (a) Find the surface charge density  $\sigma$  at  $a$  and  $b$ .  
 (b) Find the electric field  $\mathbf{E}(\mathbf{r})$  and potential  $V(\mathbf{r})$  in the regions i)  $r < R$ , ii)  $R < r < a$ , iii)  $a < r < b$  and iv)  $r > b$ .  
 (c) Find the electrostatic pressure on all surfaces at  $R$ ,  $a$  and  $b$ .  
 (d) How  $\sigma$ ,  $\mathbf{E}$ ,  $V$  and the electrostatic pressure change if the outer surface of the shell is grounded. **[2 + 14 + 6 + 2 = 24]**
3. A point charge  $q$  is situated a distance  $a$  from the center of a grounded conducting sphere of radius  $R$ . The origin is located at the center of the sphere.
- (a) Obtain an expression for the potential at any point  $V(r, \theta)$  outside the sphere in the usual spherical polar coordinates with the  $z$  axis along the line through  $q$ .  
 (b) Find the induced surface charge density  $\sigma(r, \theta)$  on the sphere, as a function of  $\theta$ . Obtain the total induced charge.  
 (c) Calculate the electrostatic energy of this configuration.  
 (d) Find the monopole moment and the dipole moment of the system. **[7 + 6 + 5 + 5 = 23]**