BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
Second Semester 2022-2023

MID-SEMESTER EXAMINATION

Course Title: Electromagnetic Theory I **Course No: PHY F212**
Max. Time: 90 mins. **Total Marks: 90** **Date: 14.03.2023.**

Instructions: Answer all questions to the point and write all parts of a single question together.

Q1. Short answer type questions: No partial marking (*bold* *faced characters represent vector*) [2×10=20M]

(a) Three point charges *q (0, 0, a), q (0, 0, -a)* and *-2q (0, a, 0)* are placed in free space (ε_0). Find the total force ***F*** *(-2q)* on the charge *-2q*.

(b) Two conducting spheres of initial charges *8q* and *–3q* respectively, are brought in contact and separated back again. If the radii of the spheres are *3r* and 2*r*, what would be the final charges on each sphere?

(c) What is the total electrostatic energy of a free point charge?

(d) If a parallel plate capacitor is filled with an insulating material of dielectric constant $\varepsilon_r = 2$, how the capacitance *C* and the electric filled ***E*** between two plates will change with respect to earlier values (vacuum)?

(e) In case of a charge distribution Q_i ($\boldsymbol{r}_i$), the total charge $\sum Q_i = 0$. If the dipole moment of this charge distribution with respect to a point *A* is $\boldsymbol{p}_a$, what will the dipole moment $\boldsymbol{p}_b$ with respect to the point *B*, which is separated by a distance ***r*** from *A*?

(f) Find the value of $\int_0^2 (x^2+3x+2)\delta(x-3)dx$

(g) For a homogeneous linear dielectric of polarization ***P*** find the value of $\oint \boldsymbol{D}.\boldsymbol{dl}$, where ***D*** represents the electric displacement.

(h) Write down the boundary conditions for the normal and the tangential component of ***E*** across a surface charge distribution with charge density 'σ'?

(i) A non-zero point charge q_1 is placed at a distance *d* in front of an infinite grounded conductor plate. Another point charge q_2 is placed at the middle (*d/2*), between the q_1 and the conductor. If the total force on q_1 is zero, find the relation between q_1 and q_2.

(j) Find the value of ∇r, where ***r*** represents the position vector.

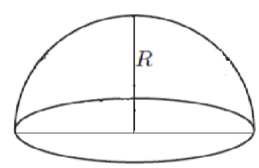


Figure 1

Q2. An inverted hemispherical bowl of radius *R* carries a uniform surface charge density *σ*, resting on XY plane and centred at the origin **(Figure 1)**.

(a) Find the potential difference between the '*north pole'* P and the *center* C.

(b) What will be the electric field $\boldsymbol{E_p}$ at the '*north pole'* P? **[9+8=17M]**

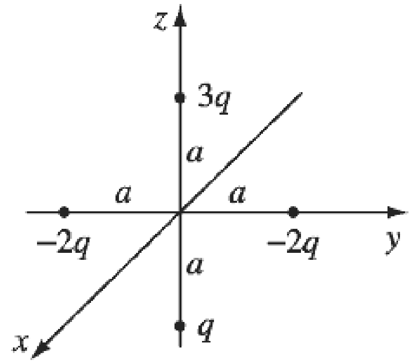
Q3. The Coulomb force ***F*** on a charge particle under an electric field ***E*** is represented in spherical polar coordinates as:

$$\mathbf{F} = k\,(r\cos\theta)\,\hat{\mathbf{r}} + (r\sin\theta)\,\hat{\boldsymbol{\theta}} + (r\sin\theta\cos\phi)\,\hat{\boldsymbol{\phi}}. \qquad (k \text{ is a constant}).$$

State the divergence theorem for the force field ***F*** and validate it within the volume of the above-mentioned inverted hemispherical shell of radius R, centred at the origin (**Figure 1**). **[15M]**

[PTO]

Q4. Four point charges 3*q (0, 0, a), q (0, 0,-a), -2q (0, a, o),* and *-2q (0,-a, 0),* are placed at distance '*a'* each from the origin, within a free space (ε_0) (**Figure 2**). Find the approximate potential $V(r,\theta,\varphi)$ at any point far from the origin, due to this charge distribution. **[12M]**



ε_1

Figure 2

Q5. If a point charge *q* is held at a point P (0, 0, d) above an infinite grounded conducting plate placed in (X-Y) plane.

a) Define the image problem with proper boundary conditions and find the potential *V (x, y, z)*, where *z* is any positive number.

b) Find the induced surface charge density *σ (x, y) and* total induced charge *Q* of the grounded plate.

c) Find the force ***F*** on the point charge *q* exerted by the grounded plate.

d) Calculate the energy *W* of this configuration. **[4+6+3+5= 16M]**

Q6. **2.** If a sphere of radius R carries a polarization $\boldsymbol{P}(\mathbf{r}) = k\mathbf{r}$

a) Calculate the bound charge densities *σ* andρ.

b) Find the electric field ***E*(r)** inside and outside of the sphere. **[5+5=10M]**

In case you may need

$$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$