Birla Institute of Technology & Science-Pilani, K. K. Birla Goa Campus

First Semester 2022-2023

PHY F213	Comprehensive Examination (Closed Book)	Maximum Marks: 50
Optics	Duration: 3 hours	Date: 17 December 2022

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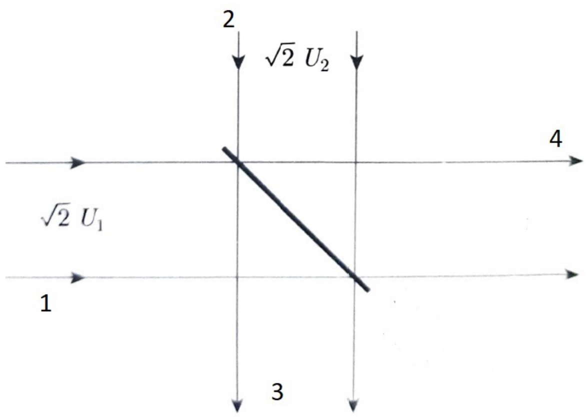
1. (a) Explain the Rayleigh criteria of limit of resolution.

   (b) The objective of a telescope has a diameter of 12.0 cm. At what distance would two small green objects 30 cm apart be just resolved by the telescope, assuming the resolution to be limited by diffraction by the objective only? Assume $\lambda = 540$ nm.

   (c) Consider a diffraction grating of width 5 cm with slits of width 0.0001 cm separated by a distance of .0002 cm. How many orders would be observable at $\lambda = 550$ nm? Would there be any missing orders? What will be the resolving power in the second order?

   [4 + 4 +4 marks]

2. Consider an ideal lossless 50% beam splitter shown in figure. Plane waves of electric field amplitude $\sqrt{2}\,|U_1|$ and $\sqrt{2}\,|U_2|$ are incident on the input ports 1 and 2 respectively. $U_1 = A_1 e^{i(k_1 z - \omega_1 t - \phi_1)}$ and $U_2 = A_1 e^{i(k_2 z - \omega_2 t - \phi_2)}$



   (a) Find the electric field and intensity in the exit ports 3 and 4, at a distance z from the beam splitter.

   (b) Discuss the interference pattern as observed by the eye if $k_1 = k_2$ and $\omega_1 = \omega_2$

   (c) Discuss the interference pattern as observed by the eye if $\omega_1 \neq \omega_2$, but $|\omega_1 - \omega_2| << 10Hz$?

   (d) Discuss the interference pattern as observed by the eye if $\omega_1 \neq \omega_2$, but $|\omega_1 - \omega_2| >> 100Hz$?

   [3 + 3 + 3 +3 marks]

3. A zone plate is to be designed and developed to have a principal focal length of 50 cm corresponding to $\lambda = 600$ nm.

   (a) Obtain an expression for the radius of the n^{th} zone?.

   (b) A point object is placed at a distance of 100 cm behind (on the axis) of the above zone plate. At what distance from the zone plate will be the image formed?

   (c) If the above zone plate is used to focus white light (wavelength between 400 nm to 600 nm), describe the focal spot. What would be the length of the focal spot ('depth of focus')?

[4 + 4 +4 marks]

4. Consider the Fraunhofer diffraction pattern of an infinitely long slit (along the y-axis) of width b (along the x-axis) observed on a screen. The screen is placed at the focal plane of a convex lens of focal length 50 cm.

   (a) When a plane wave is incident on the slit with an angle i with the normal to the plane of the slit, the diffraction pattern on the screen along the x-direction can be expressed as $I(x) = I_0(x)\frac{\sin^2\beta}{\beta^2}$. Find β (You may assume the form of the diffraction pattern corresponding to normal incidence)

   (b) When a plane wave is incident normally on the slit, show that the intensity distribution on the screen in the y-direction $I(y) \sim \delta(v)$ where $v = \frac{2\pi y}{\lambda z}$

[7 + 7 marks]

Fraunhofer Diffraction Integral

$$u(x, y, z) \approx \frac{1}{i\lambda z} e^{ikz} \exp\left\{\frac{ik}{2z}\left(x^2 + y^2\right)\right\} \times \iint A(\xi, \eta) e^{-i(u\xi + v\eta)} d\xi d\eta$$

where

$u = \frac{2\pi x}{\lambda z}$ and $v = \frac{2\pi y}{\lambda z}$