BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
First Semester 2022-2023

MID-SEMESTER EXAMINATION (Closed Book)

Optics (PHY F213)

Max. Marks: 105
Max. Time: 90 mins. **Date: 31.10.22**

Instructions:

- **Write answer all parts (and sub-parts) of the same question together.**
- **Numerical steps in problems & algebraic steps in derivations MUST be worked out and all assumptions must be stated.**
- **Draw suitable diagrams/figures, wherever required.**

GIVEN: Avogadro Number = 6×10^{23}; $\varepsilon_0 = 8.854\times10^{-12}$ C/Nm^2; m = 9.1×10^{-31} kg.; $q = 1.6\times10^{-19}$ C.

Q.1 A) Refer to the Fig. 1A, drawn below. Derive the fact that $\theta_i = \theta_r$, using the calculus version of the Fermat's principle, with θ_i as the variable.

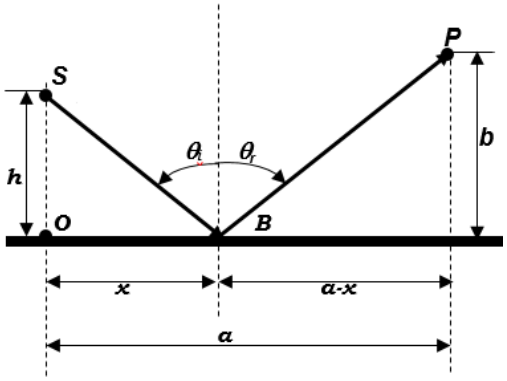

Fig. 1A

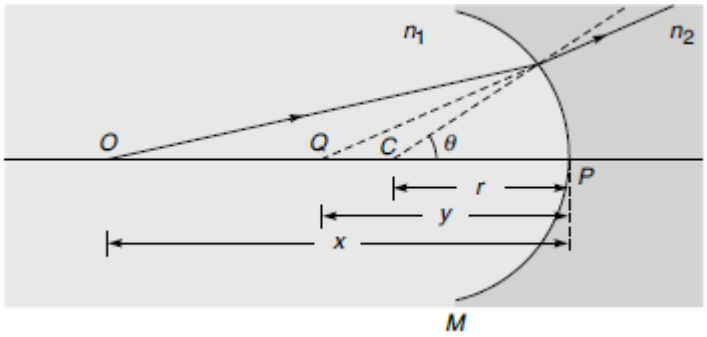

Fig. 1B

B) Consider a point object *O* in front of a concave refracting surface *SPM* separating two media of refracting indices n_1 and n_2; as shown in Fig. 1B. *C* represents the center of curvature. Write down the expression of the optical path length *Lop* in terms if *x*, *y*, *r*, θ. Show that this expression leads to the paraxial image point which is consistent with the equation $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$. *(Mention the signs of the parameters)*.

[5 + (6+4)]

Q.2 A) Consider a combination of two thin lenses of focal lengths f_1 and f_2 separated by a distance *t*. Find out the system matrix for the combination of these two lenses. Determine also the positions of the two unit planes d_{u1} and d_{u2}.

B) Refer to Fig.2B. A thick lens with equal radii of curvatures 10 cm has a thickness of 1 cm and is placed in air (refractive index = 1). An incident ray makes an angle of 10 degrees with surface 1 at an initial height of 2 mm and is transmitted after refraction from surface 2. The thick lens is made of glass with refractive index 1.5. Use matrix method to find out the final height of the refracted ray at surface 2 and the angle of the transmitted ray (in degrees) coming out at surface 2. **[(6+4) + 8]**

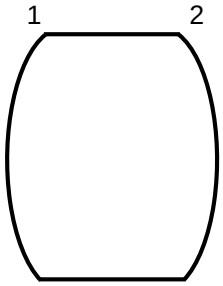

Fig. 2B

P.T.O.

Q.3 A) An achromatic cemented doublet of focal length 25 cm is made from a combination of an equiconvex flint glass lens (n_b = 1.50529, n_r = 1.49776) and a crown glass lens (n_b = 1.66270, n_r =1.64357). Find out the radii of curvatures of the different surfaces and the focal lengths of each of the two lenses.

B) Draw a diagram showing the formation of image in astigmatism, with appropriate labels. What is the measure of astigmatism? (Describe in short defining all the relevant quantities.)

[10 + (4+4)]

Q.4 A) For Sodium at λ = 1 Å, all the electrons can be assumed to be free. Under this assumption, find out the plasma frequency ω_p and the refractive index of the metal.
Given: For Na, Atomic number = 11; Density = 0.97 g/cm^3; Atomic weight = 23.

B) In a given molecule, there are f_j electrons with frequency ω_j and damping γ_j in each molecule. The refractive index of such a molecule is given by the following expression:

$$n = 1 + \frac{Nq^2}{2m\epsilon_0}\sum \frac{f_j(\omega_j^2-\omega^2)}{\left(\omega_j^2-\omega^2\right)^2+\gamma_j^2\omega^2}.$$ **(i)** Write down the wave vector k, assuming damping to be zero.

(ii) Derive an expression for the group velocity v_g and compare it with the wave velocity v.

[(5+3) + (2+8)]

Q.5 A) For a Gaussian pulse $E(z = 0, t)$ given by $E = E_0 \exp\left(-t^2/\tau_0^2\right)\exp(i\omega_0 t)$.

(i) Find out the Fourier transform of E, $A(\omega)$ as follows: $A(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} E(z=0)\exp(-i\omega t)\,dt$.

Use: $\int_{-\infty}^{+\infty} e^{-\alpha x^2+\beta x}\,dx = \sqrt{\frac{\pi}{\alpha}}e^{\beta^2/4\alpha}$.

(ii) Derive an expression for the power spectral density $S(\omega)$.

B) For pure silica, the following empirical formula may be assumed $n(\lambda_0) \approx 1.451 - 0.003\left(\lambda_0^2 - \frac{1}{\lambda_0^2}\right)$, where λ_0 is measured in μm. Calculate the zero dispersion wavelength and the material dispersion at 800 nm in ps/km.nm.

[(5+5) + (5+3)]

Q.6 A) Consider the interference pattern produced on a screen by the superposition of a plane wave incident normally and a spherical wave emanating from a point. The plane wave is given by $E_1 = E_0\, cos(kz - \omega t + \phi)$ and the spherical wave is given by $E_2 = \frac{A_0}{r}\cos(kr - \omega t)$. Derive an expression for the amplitude of the superposed wave on the screen.

B) In the Fresnel's biprism arrangement, find out an expression for d, the distance between the two virtual sources in terms of a, the distance from the source to the base of the prism; α, the angle of the biprism and n, the refractive index of the material of the prism.

[8 + 10]