Birla Institute of Technology & Science-Pilani, K. K. Birla Goa Campus

Second Semester 2022-2023

PHY F241	Comprehensive Examination (Open Text Book)	Max. Marks: 80
Electromagnetic Theory-II	Duration: 3.0 hours	Date: 11 May 2023

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1. An x-polarized monochromatic plane wave is traveling in the z-direction.

$$\mathbf{E}(z,t) = E_0 \cos(kz - \omega t + \delta)\hat{\mathbf{x}}, \quad \mathbf{B}(z,t) = \frac{1}{c} E_0 \cos(kz - \omega t + \delta)\hat{\mathbf{y}}$$

   (a) Find all elements of the Maxwell's stress tensor associated with the wave.

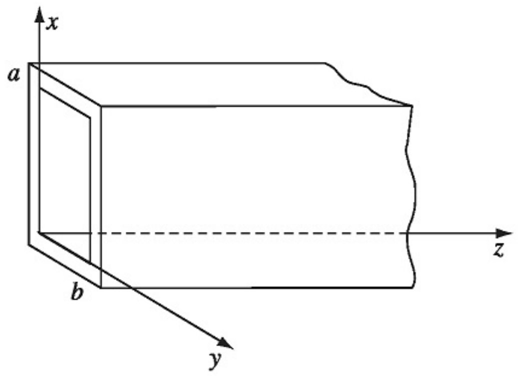
   (b) From the above, determine the momentum transported per unit area per unit time (momentum flux density) along the x, y and z directions.

   (c) Find the momentum density $\vec{\boldsymbol{g}}$ stored in the fields.

   (d) From the above, determine the momentum flux density in the direction of propagation and compare it to the energy density.

[20 marks]

2. Consider transverse magnetic (TM) waves in a rectangular wave guide (see figure) of height a and width b ($a > b$) propagating along the z-direction.



   (a) Solve the differential equation satisfied by the TM modes in a wave guide and determine the transverse electric field distribution $E_z(x, y)$ satisfying the boundary conditions.

   (b) Write down the complete expression for the electric field associated with the waves propagating along the z-direction ($\overrightarrow{\mathbf{E}}(x, y, z, t)$)

   (c) Determine the cut off frequencies. Which mode has the lowest cut-off frequency? Determine the lowest cut-off frequency.

   (d) Determine the wave velocity, the speed with which wavefront move along the z-direction

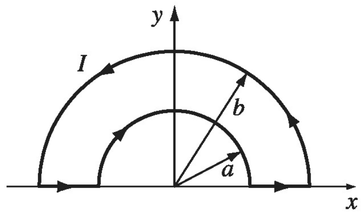
   (e) Determine the group velocity

[6+6+4+2+2 marks]

[PTO]

3. A piece of wire bent into a loop, as shown in the figure, carries a current that increases linearly with time:

$$I(t) = kt \ ; \qquad (-\infty < t < +\infty)$$



   (a) Calculate the retarded scalar potential $V(\vec{r}, t)$ at the center. [3]

   (b) Calculate the retarded vector potential $\mathrm{A}(\vec{r}, \mathrm{t})$ at the center. [6]

   (c) Does this neutral wire produce an electric field? If yes, explain why. [3]

   (d) Determine the electric field $\vec{\mathbf{E}}$ at the center. [3]

   (e) How would you determine magnetic field $\vec{\mathbf{B}}$ at the center? Write down an integral expression for $\vec{\mathbf{B}}$ at the center. [5]

[20 marks]

4. (a) Argue that the retarded vector potential in the near field, $\mathbf{A}(\mathbf{r}, t) \cong \frac{\mu_0}{4\pi r}\dot{\vec{\mathbf{p}}}\,(t_0)$ where t_0 is the retarded time at the origin (*you may use descrete charge distribution to prove your point*). From this show that $\mathbf{B}(\mathbf{r}, t) \cong -\frac{\mu_0}{4\pi rc}[\hat{\mathbf{r}} \times \ddot{\vec{\mathbf{p}}}]$.

   (*Give each step and clearly give justification for any assumptions used*)

[10 marks]

5. An insulating circular ring (radius b) lies in the $x-y$ plane, centered at the origin. It carries a linear charge density $\lambda = \lambda_0 \sin\phi$ where λ_0 is a constant and ϕ is the azimuthal angle.

   (a) Calculate the dipole moment of this charge distribution. [3]

   (b) Suppose the ring is now set to spin in the counterclockwise direction at a constant angular velocity ω. Express the dipole moment as a function of time in terms of its x and y components. [2]

   (c) Does the system under rotation emit any radiation? Why? If yes, calculate the power radiated. [5]

[10 marks]

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