Birla Institute of Technology & Science, Pilani, Pilani Campus, Rajasthan

Second Semester 2022-23 Comprehensive Examination (Open Book)

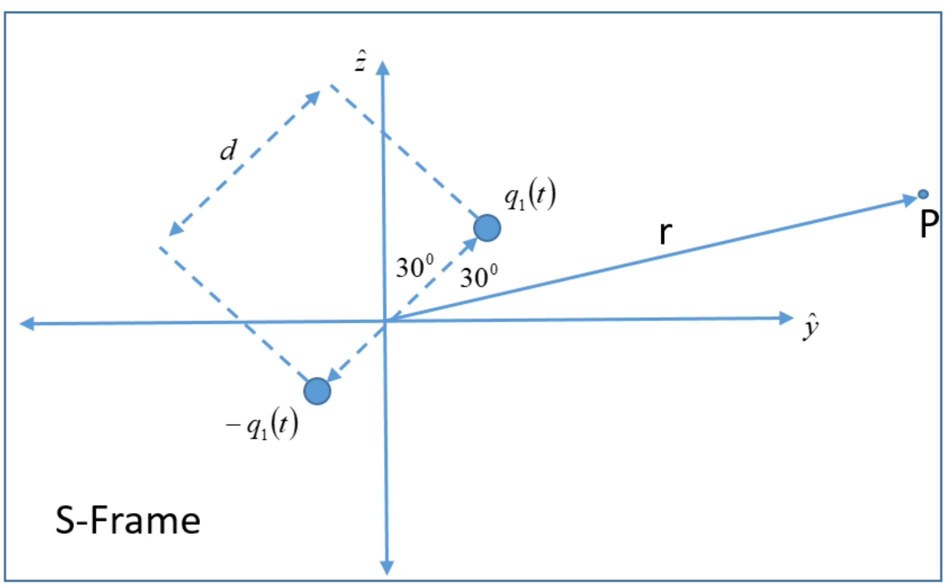
Course Title: Electromagnetic Theory II Date: 20/05/2023

Course No. PHY F241 Maximum Marks: 120 Maximum Time:180 Minutes

1. A rod of length l_0 is kept on the $x'-y'$ plane (first quadrant) of its rest frame $S'(x',y',z',t')$ with one end touching the origin O' and making an angle θ_0 with the x'-axis. Calculate the length and orientation of the same rod as observed by an observer who is in its rest frame $S(x,y,z,t)$ and the S' frame move away with respect to S with a velocity $v\hat{x}$. [10]

2. (a) Consider a particle with instantaneous acceleration $\vec{a}(a_x,a_y,a_z)$ in the $S(x,y,z,t)$. Now, evaluate the corresponding acceleration $\vec{a}'(a_{x'},a_{y'},a_{z'})$ in the $S'(x',y',z',t')$ frame which move away with respect to S with a velocity $v\hat{x}$ using Lorentz transformation. (b) From the transformation equations obtained in (a), calculate the acceleration of a particle in S frame which is instantaneously at rest in the S' frame but is accelerating at a rate $a_0\hat{x}'$ in the S' frame. [20]

3. An oscillating electric dipole as discussed in class lie on the y-z plane and is making an angle of $\theta = 30^0$ with the z-axis of the rest frame $S(x,y,z,t)$ as shown in the figure below. Calculate the expression of the (a) $\vec{E}(\vec{r},t)$, (b) $\vec{B}(\vec{r},t)$, and (c) the Poynting vector $\langle \vec{S} \rangle$ at point 'P' which is in the radiation zone and lies in the y-z plane as shown in the figure below. (Your answer should be solved in the rest frame $S(x,y,z,t)$, and you are not supposed to align the z-axis along the direction of the dipole). [20]

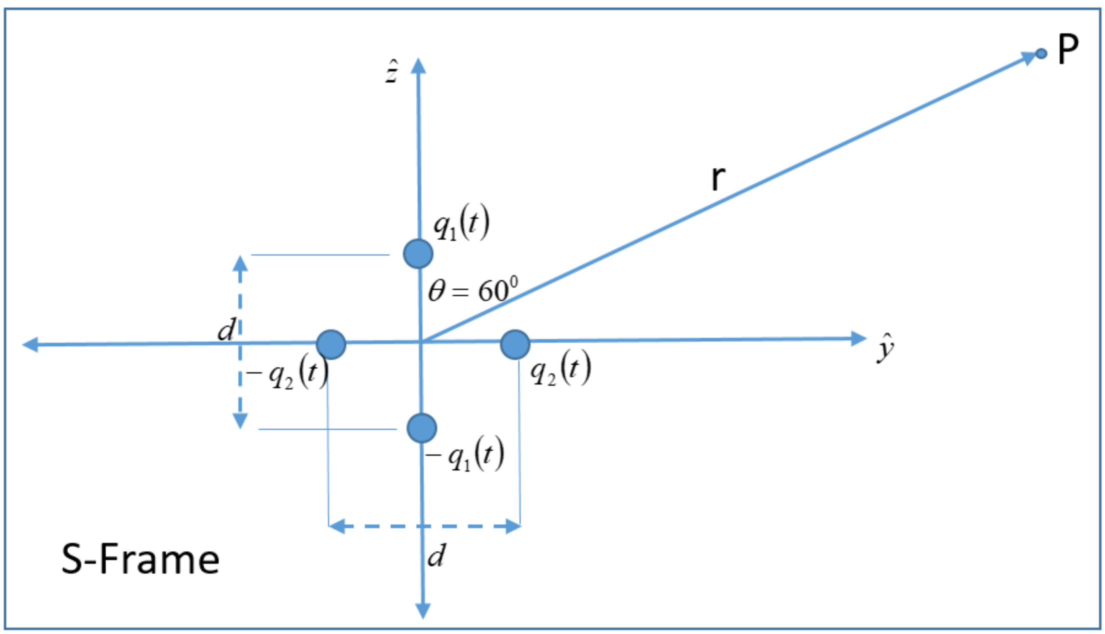


4. Two relativistic particles each having charge q move parallel to each other with the same velocity $v\hat{x}$ with respect to an inertial frame $S(x,y,z,t)$. The distance between the two charges is d. Evaluate

the expression for the force of interaction between the two charges as observed by an observer who is at rest in the frame $S(x, y, z, t)$. [20]

5. For the oscillating Magnetic Dipole problem as discussed in the class due to an oscillating circular current loop, (a) Find whether the "*Retarded Potentials*" $V(\vec{r},t)$ and $\vec{A}(\vec{r},t)$ satisfy "Coulomb Gauge" or "Lorentz Gauge" or both? (b) Also, calculate $\nabla^2\vec{A}(\vec{r},t)$ and $\frac{\partial^2\vec{A}(\vec{r},t)}{\partial t^2}$. (c) From the calculations performed in (b), show that wave equation for $\vec{A}(\vec{r},t)$ can be obtained. [20]

6. Two oscillating electric dipoles of equal magnitude, $p_0 = qd$,as discussed in the class are crossed in the y-z plane of the rest frame $S(x, y, z, t)$ as shown in figure 2 with a phase difference of $\frac{\pi}{6}$ between them. Calculate the expression of (a) $\vec{E}(\vec{r},t)$, (b) $\vec{B}(\vec{r},t)$, and (c) the Poynting vector $\langle\vec{S}\rangle$ at point 'P' which is in the radiation zone and lies on the y-z plane as shown in the figure given below. (Your answer should be solved in the rest frame $S(x, y, z, t)$) [20]



7. The electric field in the radiation zone for a certain configuration is given as

$$\vec{E}(r,\theta,\varphi,t) = \frac{k^2 p_0}{4\pi\varepsilon_0}\left(\cos\theta\hat{\theta} + i\hat{\varphi}\right)\frac{e^{i(kr-\omega t+\varphi)}}{r}.$$

From the above expression, extract the (real) electric fields on the positive x-,y- and z- axes. [10]