Birla Institute of Technology and Science, Pilani

Second Semester 2022-23 Mid-Semester Examination (Open Book)

Course Title: Electromagnetic Theory II **Date: 18/03/2023**

Maximum Marks: 90 **Maximum Time: 90 mins**

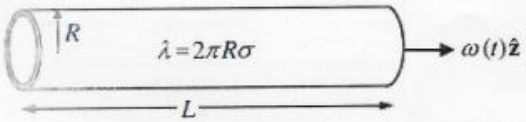
1. The angular velocity $\omega(t)\hat{z}$ of the cylindrical shell shown below increases from zero and smoothly approaches the steady value ω_0. The shell has infinitesimal thickness and carries a uniform charge per unit length $\lambda = 2\pi R\sigma$, where σ is a uniform charge per unit area. Assume that the shell radius R << L and that the spin-up is very slow so the displacement current may be neglected.

(a) Find the static electric field and steady magnetic field when $\omega = \omega_0$.

(b) Find $B(\vec{r},t)$ everywhere during the spin-up and use it to find the time-dependent part of the electric field.

(c) The spin-up is performed by an external agent, who supplies power at a rate $-\vec{J} \bullet \vec{E}$ per unit volume to create the magnetic field. Confirm this by evaluating Poynting's theorem over all of space

(d) Evaluate Poynting's theorem using a cylindrical volume with a radius slightly smaller than the shell to study the flow of energy into the interior from the surface of the shell.



[5+5+5+5]

2. Let $E(z,t) = E_0 \cos(kz - \omega t)\hat{x} + E_0 \cos(kz + \omega t)\hat{x}$ represent two counter-propagating plane waves.

(a) Write $E(z,t)$ in a simpler and compact form and find the associated magnetic field $B(z,t)$.

(b) For the fields in part (a), find the instantaneous and time-averaged electric and magnetic field energy densities. [5+5]

3. For the rectangular wave-guide ($a = 3.0$ cm and $b = 1.5$ cm) discussed in class,

(a) Calculate the cut-off frequency for TE_{21} mode?

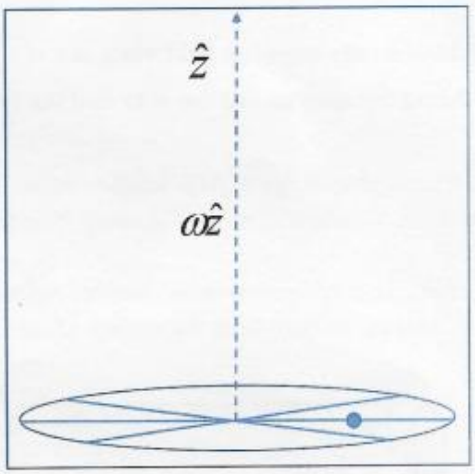
(b) Write the expression for $B_x(x,y)$, $B_y(x,y)$, and $B_z(x,y)$ for TE_{32} mode.

(c) Write the expression for $E_x(x,y)$, $E_y(x,y)$ and for TE_{42} mode.

(d) Write the generic expression for the total electric field ($\vec{\tilde{E}}(x,y,z,t)$) and the total magnetic field ($\vec{\tilde{B}}(x,y,z,t)$) for TE_{56} mode. [6+6+4+4]

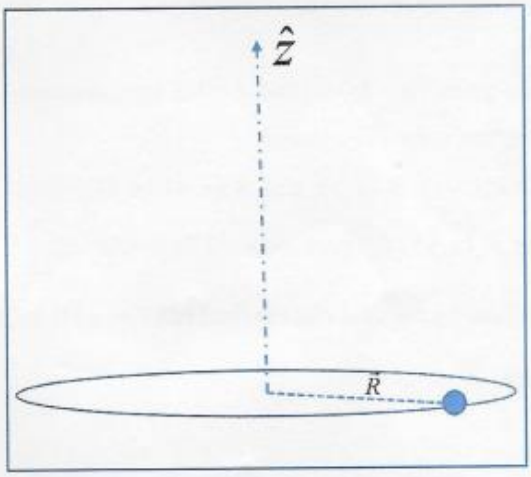
4. A bead of mass m and charge q moves along the spoke (non-conducting) of a wheel with a constant speed u meters per sec. The wheel rotates with uniform angular velocity $\dot{\theta} = \omega$ radians per sec about an axis fixed in space. At t=0, the spoke is along the x-axis, and the bead is at the origin. Find the Lienard-Wichert potentials, $V(r,t)$ and $A(r,t)$ for the points on the z-axis.

[10+10]



5. A bead of mass m and charge q moves freely with a constant speed u on a non-conducting circular wire of radius R which is kept on the x-y plane in free space (Neglect gravity). At t=0, the bead's position is (R,0). Calculate the expression for the $\vec{E}(r,t)$ and $\vec{B}(r,t)$ at the origin of the circular wire.

[10+10]



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