Physics Department, BITS - Pilani, Pilani ; 2nd Sem' 2016-17

Comprehensive Examination

QUANTUM MECHANICS I (PHY F242) ; PART - A (CB)

Date : May 8, 2017 Max. Time : 3 hrs (Part - A + Part - B) Max. Marks : 40

Note : Label the parts [like 1(a), 2(b) etc.] of the questions properly. Write all parts of a question together.

Q1.(a) In a double-slit experiment, a beam of electrons directed towards a plate containing two slits (say, A & B). Beyond that plate, there is a vertical screen (say, along y-axis) which monitors the diffraction pattern on the screen. When slit A is open, the amplitude of the electrons detected on the screen is $\psi_A(y) = \frac{1}{\sqrt{\pi(1+y^2)}}e^{-iky}$. When B is open, it is $\psi_B(y) = \frac{1}{\sqrt{\pi(1+y^2)}}e^{-i(ky+\pi y)}$. Calculate and plot the intensity pattern on the screen when **(i)** both slits are open and a light source is used to determine which of the slits the electrons passed through and **(ii)** both slits are open and no light source is used. [**3 + 3**]

Q1.(b) Consider a particle of mass m and energy E moving towards a barrier,

$$V(x) = 0 \quad ; x < 0;$$
$$= V_0 \quad ; 0 \leq x \leq a;$$
$$= 0 \quad ; x > a \; .$$

Plot the probability density $|\psi(x)|^2$ of the particle at various regions, when $E > V_0$ and $E < V_0$. [**2 + 2**]

Q1.(c) For parity ($\hat{P}$), prove/disprove that it is **(i)** hermitian and **(ii)** commutes with momentum $\hat{p}$. [**2 + 2**]

Q2. A charged oscillator of mass m, charge q and electric field E_0 is described by the Hamiltonian, $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 + qE_0\hat{x}\; cos\omega t$. Use Ehrenfest equations to determine $< x >_t$ with the initial condition, $< x >_{t=0} = x_0$. [**4**]

Q3. Consider a particle of mass m and energy E ($E > V_0$) moving towards a step potential,

$$V(x) = 0 \quad ; x < 0;$$
$$= V_0 \quad ; x \geq 0.$$

Calculate the reflection coefficient (R) and transmission coefficient (T) as a function energy E of the particle. Express your final answer in terms of α $(= \sqrt{1 - \frac{V_0}{E}})$. Plot R and T vs. E. In the graph, show the values of R and T at two extremes, i.e., at $E \simeq V_0$ and $E >> V_0$. [**6**]

Q4. A quantum oscillator with Hamiltonian, $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$ is in a state, $\psi(x,0) = (\alpha/5)(1-2\sqrt{\frac{m\omega}{\hbar}}x)^2 exp(-\frac{m\omega x^2}{2\hbar})$. Where, $\alpha = (\frac{m\omega}{\pi\hbar})^{1/4}$. **(a)** Determine the average parity $< \hat{P} >$. **(b)** At time t, the wave function becomes $\psi(x,t) \propto (1+2\sqrt{\frac{m\omega}{\hbar}}x)^2\; exp(-\frac{m\omega x^2}{2\hbar})$. What is the smallest possible value of t (in terms of ω) for the above evolution to take place ? [*1st three wave functions of H.O. :* $\phi_0 = \alpha e^{-\rho^2/2}$, $\phi_1 = \alpha\sqrt{2}\rho e^{-\rho^2/2}$ *and* $\phi_2 = \frac{\alpha}{\sqrt{2}}(2\rho^2 - 1)e^{-\rho^2/2}$; $\rho = x/a_0$; $a_0 = \sqrt{\frac{\hbar}{m\omega}}$.] [**6 + 4**]

Q5. A particle of mass m in the infinite well of width L starts out in the left half of the well and equally like to be found at any point in that region. What is the probability that measurement of the energy yields the ground state energy ? [*Energy eigen functions of the system :* $\phi_n(x) = \sqrt{2/L}\; sin(n\pi x/L)$; $0 \geq x \leq L$.] [**6**]