Physics Department, BITS - Pilani, Pilani ; 2nd Sem' 2016-17

Mid Semester Examination (CB)

QUANTUM MECHANICS I (PHY F242)

Date : 8th March 2017 Max. Time : 1.5 hrs Max. Marks : 60

Q1.(No quantum effects in macroscopic world !) Consider a (genuine) fast bowler bowls with average speed 150 km/hr (take the mass of the cricket ball as 150 gm). Calculate the associated de Broglie wave length λ_B and the Compton wave length λ_c of the ball. How large/small are these values in compared to the size of a proton ($\sim 1\ fm$). Do the order of magnitude estimate only. [**4**]

Q2. (classical variables $\Rightarrow$ quantum operators) For the wave function $\psi(x) = 3x^2\ exp(-\frac{iEt}{\hbar})$, calculate the commutator $[\hat{x},\ \hat{p}^2]\ \psi(x)$. [**4**]

Q3. (special op. $\Rightarrow$ physical observables) For any arbitrary operator $\hat{A}$ and its adjoint $\hat{A}^\dagger$ (not necessarily self-adjoint), which among the following combinations will give purely real/imaginary eigen values. Justify your answer (in one/two sentences). **(a)** $\hat{A} + \hat{A}^\dagger$ **(b)** $\hat{A} - \hat{A}^\dagger$ **(c)** $i\ (\hat{A} + \hat{A}^\dagger)$ **(d)** $i\ (\hat{A} - \hat{A}^\dagger)$. [**4**]

Q4. (uncertainty) Consider the following set of wave functions $\psi(x)$ in coordinate (1-d) representation, determine the corresponding wave function $\phi(p)$ in momentum representation. Plot (the real part of) $\psi(x)$ and corresponding $\phi(p)$. Comment on uncertainty principle based on your results. The systems are considered to be extended over all space. **(a)** $\psi(x) = |A|\ exp\ (\frac{ip_0 x}{\hbar})$ **(b)** $\psi(x) = |A|\ \delta(x)$. [**7 + 7**]

Q5.(measurement) Here is a simple eigenvalue problem, $i\frac{df(\phi)}{d\phi} = \lambda f(\phi)$, where ϕ is the standard azimuthal angle and $f(\phi) = f(\phi + 2\pi)$. **(a)** Determine the set $\{\lambda_n\}$ and the corresponding normalized set of eigenfunctions $\{f_n(\phi)\}$. **(b)** Show that $\{f_n(\phi)\}$ form an orthonormal basis. **(c)** If the state of the system is described by $F(\phi) = \sqrt{\frac{4}{3\pi}}\ cos^2\phi$ (the prefactor ensures normalized $F(\phi)$), determine the probable outcomes (i.e., various eigenvalues λ_n) of measurement and the probability P_n of each outcome (include non-zero probability only). [**6 + 4 + 7**]

Q6. (Eigen function, uncertainty and evolution of w.f.) You have solved the "free particle in a 1-d box" problem when the walls of the box are at x = 0 and x = L. The resulting energy eigenfunctions are given as, $\phi_n = \sqrt{\frac{2}{L}}\ sin\ \frac{n\pi x}{L}$ with corresponding eigenvalues E_n, where n = 1,2,3,... Now, if the walls are at $x = \pm L/2$, **(a)** obtain the energy eigen function (by inspection/symmetry argument from above result), **(b)** for the lowest state, determine the standard deviations Δx & Δp and hence the product $\Delta x\ \Delta p$. **(c)** Suppose that at t = 0, the state of the particle is described by $\psi(x,0) = \frac{3\phi_1 + 4\phi_5}{5}$, write down the state $\psi(x,t)$ at later time t. [**5 + 10 + 2**]

Useful constants and formulae : $c = 3 \times 10^8$ m/s, $h = 6.63 \times 10^{-34}$ j-s

Fourier transform : $\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \phi(p)\ e^{\frac{ipx}{\hbar}}\ dp$; $\phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi(x)\ e^{\frac{-ipx}{\hbar}}\ dx$; $\delta(p) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} e^{\frac{-ipx}{\hbar}}\ dx$

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