BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

Second Semester, 2022-2023

Comprehensive Examination: Part B - Open Book

Quantum Mechanics-1 (PHY F242)

Date: 15.05.2023 **Time:** 90 min **Maximum Marks:** 60

1. (a) (4) Classical wave optics admits a relation $\Delta x \Delta k \geq \frac{1}{2}$, where symbols have their standard meaning. Explain its physical significance.
(b) (4) If you multiply both sides of this equation by $\hbar$, it implies Heisenberg's uncertainty principle. This seems to imply that Heisenberg's uncertainty principle is already contained in classical physics. Provide an argument for or against this case, as you deem appropriate.
(c) (7) Obtain the ground state energy for a quantum harmonic oscillator using Heisenberg's uncertainty principle.

2. Treating X and P_x as non-commuting operators corresponding to the x-component of position and momentum, prove the following identities: [5+7]
(a) (5) $[X^n, P_x] = in\hbar X^{n-1}$
(b) (7) $[XP_x, H] = X[P_x, H] + [X, H]P_x$

3. Consider a particle of m subject to an attractive delta function potential $V(x) = -\lambda\delta(x)$. Here λ is a constant that characterises the strength of the potential. If there is a bound state, its energy eigenvalue would be negative because potential is zero in the wings of the potential energy curve.
(a) (8) Obtain the wavefunctions in the region $x < 0$ and $x > 0$ and determine the constant of proportionality for the wavefunctions in terms of the parameters of the problem.
(b) (7) Obtain the energy eigenvalue(s).

4. Show that for a general one-dimensional free-particle wave-packet,
(a) (7) the expectation value $\langle p \rangle_x$ does not change with time. [7]
(b) (6+2) the expectation value $\langle x \rangle = \langle x \rangle_{t=t_0} + \frac{\langle p \rangle_x}{m}(t - t_0)$. What is the physical significance of this result?