BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
Second Semester, 2022-2023
Mid-Semester Examination (Closed Book)
Quantum Mechanics-1 (PHY F242)

Date: 16.03.2023 **Time:** 90 min **Maximum Marks.:** 100

Some constants you may need:

$h = 6.626 \times 10^{-34} Js$; $c = 3 \times 10^8 m/s$; $1\ eV = 1.6 \times 10^{-19} J$

1. Obtain the expression for average energy per oscillatory degree of freedom due to Planck by considering discrete quantum for energy. [20]

2. (a) Light from a sodium vapour lamp has a wavelength of 589 nm. What is the energy of the photon (in ev) corresponding to this wavelength?
(b) Electric field of radiation is given as $\vec{E} = E_0 \hat{i} \cos[k_0(y+z) - \omega t]$. What is the magnitude and direction of the photon momentum?

   (c) The work function of a metal is $2.0 \times 10^{-19} J$. (i) What is the threshold frequency for photoelectric emission? (ii) If the metal is exposed to a light beam of frequency $6.0 \times 10^{14}\ Hz$, what will be the stopping potential in volts (potential required to stop the emission of electron)? [10]

3. (a) Whenever $\int_V |\psi|^2 d\tau \neq 1$ you need to normalise the wavefunction. Normalise the wavefunction $\psi(x) = e^{-|x|} \sin \alpha x$. [10]
(c) If a particle is described by the above normalized wavefunction, what is the probability that its position is to the right of the point $x = 1$? [10]

4. Consider a wave-packet $\Psi(x,t)$ formed by super-posing plane-waves of amplitude $\phi(p_x)$ in momentum space.Assume that $\phi(p_x)$, is sharply peaked at $p_x = p_0$ and falls rapidly to zero outside the interval $(p_0 - \Delta p_x, p_0 + \Delta p_x)$. Show that such a wave-packet is the product of a plane wave of wavelength $\lambda_0 = h/p_0$ and angular frequency $\omega_0 = E(p_0)/\hbar$ times a modulating envelop function $F(x,t)$, and that this envelop function propagates without change of shape with a group velocity $v_g = p_0/m$, as long as $\frac{(\Delta p_x)^2 t}{2m\hbar} << 1$. [20]

5. Obtain the Bohr frequency relation ν_{ab} corresponding to transition between two energy levels E_a and E_b.Show that using Bohr's correspondance principle, it reproduces the frequency expected in classical physics. [20]