Birla Institute of Technology and Science - Pilani, Pilani Campus
Semester II (Session 2022-23)
Midsemester Examination (Closed Book)
Mathematical Method of Physics (PHYF243)

Date : 14/03/2023 Weightage : 30 %
Time: 90 Mints. Max. Marks: 90

Q1: (i) Write transformation rules for mixed tensor; A^{klm}_{ij} and covariant vector B_n. Determine the tensor inner product taking $k = n$. (ii) Assume a mixed tensor C^{ijk}_{lm} of rank 5. Do all possible contraction operation and find out the rank of the resulting tensor. [10+5]

Q2: Assume 4-D Cartesian Minkowski coordinate system (ct, x, y, z). Write down covariant basis vectors in corresponding 4-D Minkowski spherical polar coordinate (ct, r, θ, ϕ) system using Cartesian Minkowski coordinates basis functions, i.e.,$\hat{e_t}$, $\hat{e_x}$, $\hat{e_y}$, $\hat{e_z}$. Using above basis vectors, write covariant metric tensor in 4-D Minkowski spherical polar coordinate system. [Use $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$.] [15]

Q3: (i) In a vector space of functions, scalar product is defined by $< f|g >= \int f^*(x)g(x)dx$. Expand $\exp(2\pi x)$ in terms of a orthogonal, but non-normalized set of basis functions $\chi_0 = 1$, $\chi_1 = \sin x$, $\chi_2 = \sin 2x$, $\chi_3 = \sin 3x$ in the range $(0, \pi)$ and determine the coefficients required for the expansion. [You can leave your answer in terms of integral !]. (ii) The value of covariant metric tensor in 3-D circular cylindrical polar coordinate system (ρ, ϕ, z) as $g_{\rho\rho} = 1$, $g_{\phi\phi} = \rho^2, g_{zz} = 1$. Find the value of Christoffel's symbol Γ^3_{31}. [10+5]

Q4: Use Gram-Schmidt orthogonalization process to find first three orthonormalized set of functions of a vector space of functions defined over a range $[-1, 1]$ and spanned by the basis functions $\phi_0(x) = 1$, $\phi_1(x) = x$, $\phi_2(x) = x^2$, $\phi_3(x) = x^3$. [Use scalar product defined in Q3]. [15]

Q5: (i) A vector $\vec{V}$ is expressed as; $\vec{V} = V^i\vec{\epsilon_i}$ (with $\vec{\epsilon_i}$ as basis vector) in any non-orthogonal coordinate system say (q^1, q^2, q^3). Obtain its covariant derivative in terms of Christoffel's symbol and express total derivative of $\vec{V}$ i.e., $d\vec{V}$ in terms of it. (ii) Show that $\Gamma^i_{jk} = \Gamma^i_{kj}$. [10+5]

Q6: (i) Express $\frac{\partial\vec{\epsilon^i}}{\partial q^j}$ in terms of Christoffel's Symbols (sometimes called Omega coefficients). (ii) For the transformation $u(x, y, z) = x + y - z$, $v(x, y, z) = x/y$, $w(x, y, z) = x - y + z$ with $x >= 0, y >= 0, z >= 0$, find the Jacobian $J = \frac{\partial(x,y,z)}{\partial(u,v,w)}$. [10+5]

**** Best Wishes ****