Birla Institute of Technology and Science - Pilani, Pilani Campus

Semester II (Session 2022-23)

Comprehensive Examination

Mathematical Method of Physics (Closed Book)

Date : 11/05/2023 | Weightage : 20 %

Time: 90 Mints. | Max. Marks: 60

Q1: (i) Assuming a 3-D spherical polar coordinate system, define three basis vectors, $\vec{\epsilon}_r$, $\vec{\epsilon}_\theta$ and $\vec{\epsilon}_\phi$. Using above basis vectors, obtain factors h_r, h_θ and h_ϕ. Also find the matrix representation of the covariant metric tensor g_{ij} in spherical polar coordinate system.
(ii) Define Christoffel's symbol Γ^i_{jk}. Express it in terms of derivatives of the metric tensors. [6+6]

Q2: (i) If $A_k = \frac{1}{2}\epsilon_{ijk}B^{ij}$, where B^{ij} is an antisymmetric tensor. Show that; $B^{mn} = \epsilon^{mnk}A_k$.
(ii) A 2-D orthogonal system is described by the coordinates q_1 and q_2. Show that the Jacobian J transforming (x, y) to (q_1, q_2) satisfies the equation:

$$J = \frac{\partial(x,y)}{\partial(q_1,q_2)} = \frac{\partial x}{\partial q_1}\frac{\partial y}{\partial q_2} - \frac{\partial x}{\partial q_2}\frac{\partial y}{\partial q_1} = h_1\, h_2, \tag{1}$$

where $h_i^2 = g_{ii}$. [6+6]

Q3: Assuming ψ which is expressed in terms of orthonormal basis functions $|\phi_\mu >$ by using coefficients c_μ as; $\psi = \Sigma_\mu c_\mu \phi_\mu$. Define an unitary matrix $u_{\nu\mu}$ that connects old basis $|\phi_\mu >$ to the new basis $|\phi'_\nu >$. (i) Use above two equations to write down matrix representation of unitary matrix $u_{\nu\mu}$ in terms of the basis functions. (ii) Show that new coefficient vector c' in new basis functions is related to the old coefficient vector c in old basis functions by the matrix equation: $c' = Uc$, where U is unitary matrix. [6+6]

Q4: Solve boundary value problem; $y'' = \sin x$, $y'(0) = 0$; $y(\pi) = 0$. [12]

Q5: (i) Find the residue of the function $f(z) = \frac{\sinh z}{z^2 \cosh z}$ at zero of $\cosh z$ i.e., at $z = \pi i/2$.
(ii) Evaluate $\int_0^\infty \frac{\ln x\, dx}{x^2+4}$ using the method of Complex integration. [3+9]

**** Best Wishes ****

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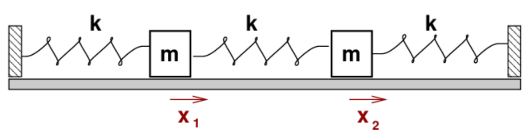
Q1: A_{ij} in 3-D coordinate system can be expressed as:

$$A_{ij} = \begin{bmatrix} a & b & c \\ 0 & d & e \\ -d & 0 & f \end{bmatrix}$$

Calculate the components of the dual tensor defined by; $V^{ij} = \frac{1}{2}\epsilon^{ijkl}A_{kl}$. [12]

Q2: (i) Find the value of the following Christoffel symbols of the second and first kind in cylindrical coordinates: Γ^2_{21}, $[12, 1]$.
(ii) Find the Christoffel symbol of the second kind Γ^1_{21} in parabolic coordinate system; $(x^1, x^2, x^3) = (\xi, \eta, \phi)$, where $x = \xi\,\eta\cos\phi$, $y = \xi\,\eta\sin\phi$, $z = \frac{1}{2}(\xi^2 - \eta^2)$. [12]

Q3: (i) Use the eigen value method to determine the normal modes of the coupled oscillator shown in Figure below (assume no friction between mass and table); Also determine the corresponding eigen vectors.



(ii) Diagonalize the following matrix by using unitary transformation, if possible:

$$B = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

[12]

Q4: Find and use the initial value Green's function to solve the ODE; (i) $x^2y'' + 3xy' - 15y = x^4e^x$, $y(1) = 1; y'(1) = 0$. [12]

Q5: Evaluate the integral using the method of Complex Integration: $\int_0^{2\pi} \frac{\cos 2\theta\, d\theta}{1-2a\cos\theta+a^2}$; $-1 < a < 1$. [12]

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