Physics Department, BITS - Pilani, Pilani ; 1st Semester 2016 -2017

Statistical Mechanics (PHY F312); Comprehensive Examination

Date : 9/12/16 ; Time: 9 AM - 12 PM ; Max. Marks : Part - A (40) + Part - B (40)

PART - A (Close Book)

Instruction : Each answer should be supplemented by few relevant mathematical steps. You are recommended to finish this part in 1 hr 15 min. Collect Part - B (open book) only after submitting this part. Standard/lecture class symbols have been used. Question paper is printed on both sides.

Q1. (Equipartition). If c_P & c_V are the specific heat at constant pressure and constant volume respectively, use equipartition theorem to determine the ratio $\frac{c_P}{c_V}$ for (a) d-dimensional classical harmonic oscillator and (b) for a classical (non-relativistic) ideal gas in d-dimensions. [**2 + 2**]

Q2. (M-B) (a) Consider an ideal gas of atoms (mass m of each atom) at temperature T, which follows M-B velocity distribution, $f(\vec{v})d^3v \propto e^{-\frac{m\vec{v}^2}{2k_BT}}d^3v$. Find the most probable value of the kinetic energy ϵ_p.

(b) N weakly coupled classical particles (distinguishable) obeying M-B statistics may each exist in one of the 3 non-degenerate energy levels of energies -E, 0, & +E. The system is in contact with a thermal reservoir at temperature T. What is the maximum possible entropy of the system ? [**4 + 2**]

Q3. (F-D & B-E) Consider a collection of two identical non-interacting particles occupying the single particle energy levels, $0, \epsilon$ & 2ϵ. Total energy of the system is fixed at $E = 2\epsilon$. Use MCE to determine the entropy of the system, if the particles follow (a) B-E and (b) F-D statistics. Ignore spin degeneracy. [**2 + 2**]

Q4. (1-d quantum oscillator) A simple harmonic 1-d oscillator has energy levels $E_n = (n+1/2)\hbar\,\omega;\ n = 0, 1, 2, ...$ The oscillator is in thermal contact with a heat reservoir at temperature T. Within canonical ensemble, find the average energy $< E >$ of the oscillator as a function of temperature T. [**4**]

Q5. (Black body) Measurement of spectral distribution from a certain star (assume sphere of radius R_{star}) indicates that the stars surface temperature is 3000K. The star is also found to radiate 100 times the power radiated by Sun (radius R_{sun}). Determine the ratio $\frac{R_{star}}{R_{sun}}$. Take surface temperature of the Sun as 6000 K. [**4**]

Q6. (Pauli's paramagnetism) Consider a non-interacting quantum gas of fermions with spin s = 1/2 at $T = 0$. In a uniform magnetic field H along z-direction, the single particle energies become, $\epsilon = \frac{p^2}{2m} - 2\mu_0 H m_s$, where $m_s = \pm 1/2$ and μ_0 is a constant with the units of magnetic moment. For N particles in a volume V, find an expression for the minimum field H_0 that will give rise to total polarization of the spins, i.e., is no particles left with the high energy spin orientation. [**6**]

Q7. (White dwarf) A white-dwarf is thought to constitute a degenerate electron gas system at a uniform temperature much below the Fermi temperature. This system is stable against gravitational collapse so long as the electrons are non-relativistic. Calculate the typical electron density (numerical value in m^{-3}) for which the Fermi momentum is one-tenth of $m_e c$. (m_e is electron's rest mass, c is the speed of light in vacuum.) [**6**]

Q8. (CMBR) Early in the evolution when the universe occupied a much smaller volume and was very hot, matter and radiation were in thermal equilibrium. When the temperature fell to about 3000K, matter and the cosmic radiation (black body) became decoupled. The temperature of the cosmic radiation has been measured to be 3K (approx.) now. What is the number density (numerical value in cm^{-3}) of CMBR photon today ? [**6**]

You may use :

Riemann zeta function $\zeta(\nu) = \frac{1}{\Gamma(\nu)} \int_0^\infty \frac{x^{\nu-1}}{e^x - 1}\, dx$; $\zeta(3) \simeq 1.2$; Planck's constant $h = 4.14 \times 10^{-15}$ eV - s ; Boltzmann constant $k_B = 8.62 \times 10^{-5}$ eV K^{-1}; mass of electron $m_e = 9.1 \times 10^{-28}$ gm .
