Physics Department, BITS - Pilani, Pilani ; 1st Semester 2016 -2017

Statistical Mechanics (PHY F312); Comprehensive Examination

Date : 9/12/16 ; Time: 9 AM - 12 PM ; Max. Marks : Part - A (40) + Part - B (40)

PART - B (Open Book)

Note : For this part, only class notes (hand written and/or photocopy) and text book (Pathria) is allowed. No derivations are required for the expressions which appear in the text book and/or expected to be there in the class notes. You can directly copy them from the source.

Q1. (M-B statistics) A gas of N classical, non-relativistic, non-interacting atoms (of mass m each) is held in a neutral atom trap by a potential of the form $V(r) = cr$, where c is a constant and $r = (x^2 + y^2 + z^2)^{1/2}$. The gas is in thermal equilibrium at a temperature T. (a) Find the single particle canonical partition function $Q_1(T)$ of the system. Express your answer in the form $Q_1(T) = AT^{\alpha}c^{-\beta}$ and find the prefactor A, the exponents α & β. [*Hint: For potential part, express & perform the integration in spherical coordinates.*] (b) Find the entropy $S(N, Q_1)$ of the gas in terms of N & Q_1 with & without Gibbs correction (N!) factor. Check the extensive property of entropy and justify the inclusion/exclusion of Gibbs correction. (c) Find the pressure P of the gas. [**6 + 4 + 2**]

Q2. (T - P distribution) Consider a system described by an ensemble with partition function $J(T, P, N) = \int_0^\infty e^{-\beta PV} Q(T, V, N)\, dV$ and corresponding thermodynamic potential $G(T, P, N)$. Where, $Q(T, V, N)$ is the canonical partition function of the system. (a) Do the necessary thermodynamic steps to relate G(T,P,N) with J(T,P,N). (b) Now consider a non-relativistic classical (indistinguishable) ideal gas at a temperature T, pressure P. Determine the partition function J(T,P,N) of the system. You can copy the expression of Q(T,V,N) from the text book/class notes. (c) Determine the pressure P(T,V,N) of the gas. [**4 + 6 + 2**]

Q3. (d-dimensional universe) Assume a d-dimensional expanding (*adiabatically*) universe filled with (black body) radiation (photon) only. The expansion is characterized by "radius" R (volume $V \propto R^d$). The energy density of the radiation varies with temperature as $\rho(T) \propto T^{\alpha}$ and the equation of state is given as $P = \omega\rho$. (a) Determine α and ω for the d-dimensional universe (*no need to carry out any integration*). (b) If R varies with temperature as $R \propto T^{\delta}$, find δ as well for this d-dimensional universe. [**6 + 4**]

Q4. (Fermions at zero chemical potential) Consider a Fermi system of an ideal gas with fixed number density. Now, define a temperature T_0 at which the chemical potential of the system is zero. (a) Express T_0 in terms of Fermi temperature T_F of the gas and Riemann zeta function $\zeta(\nu)$. (b) For $T_F = 10^4$ K, calculate the numerical value of T_0 (in K). [**4 + 2**]

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