Birla Institute of Technology & Science, Pilani

Semester I (Session 2016-17)

Mid Semester Examination (CB)

STATISTICAL MECHANICS

Max. Time: 1.5 hrs **Max. Marks : 60**

Q1. Suppose a thermal system of N independent particles in d-dimensions at a fixed temperature T. The single particle Hamiltonian of the system is given as, $h = A|\vec{p}|^m + B|\vec{q}|^n$. Compute the (ensemble) average energy $<\epsilon>$ of each particle by applying equipartition theorem, $< x_i \frac{\partial h}{\partial x_j} >= \delta_{ij} KT$. Where x_i denotes the coordinates q & p. **[10]**

Q2. Consider a system of N (distinguishable) particles, where the possible energy values for a single particle are $0, \epsilon_1, ..., \epsilon_n$. The energy value ϵ_a is g_a - fold degenerate, and the ground state is non-degenerate. (i) If $p_0(T)$ is the probability for a single particle to be in the ground state, determine $p_0(T \to \infty)$ and $p_0(T \to 0)$. (ii) Calculate average energy $<\epsilon>$ per particle at $T \to \infty$. **[5 + 3]**

Q3. N diatomic (distinguishable) molecules are stuck on a metal surface. Each molecule can either lie flat on the surface, in which case it must be aligned to one of two directions, $+x$ and $+y$, or it can stand up along the $+z$ direction. There is an energy cost of ϵ (> 0) associated with a molecule standing up, and zero energy for molecules lying flat along x or y directions. Suppose N_z number of molecules stand up along z-direction at a temperature T. Use microcanonical ensemble to determine the ratio of $\frac{N_z}{N}$ for which entropy of the system becomes maximum. (*Use Stirling approximation,* $N! = N\ ln\ N - N$.) **[12]**

Q4. Consider a rod-shaped diatomic molecule with moment of inertia I, and a (electric) dipole moment μ. The contribution of the rotational degrees of freedom to the Hamiltonian is given by, $H = \frac{p_\theta^2}{2I} + \frac{p_\phi^2}{2I\ sin^2\theta} - \mu E cos\theta$. Where E is an external electric field. θ & ϕ are the usual polar and azimuthal angles, receptively, while p_θ & p_ϕ are their conjugate momenta. (a) Calculate the classical partition function $Q_1(T)$ of each dipole. (b) Obtain the average polarization, $P =< \mu cos\theta >$ of each dipole. (*You can use,* $\int_0^\infty e^{-cx^2} dx = \frac{\sqrt{\pi}}{2\sqrt{c}}$; $\int_0^\infty e^{-x} x^{n-1} dx = \Gamma(n)$; $\Gamma(1/2) = \sqrt{\pi}$.) **[12 + 6]**

Q5. N (non-interacting) hydrogen molecules H_2 absorbed on a (two-dimensional) flat surface are in thermal equilibrium at temperature T. The rotational motion of each molecule is confined to the plane of the surface. The quantum state of the planar rotation is specified by a single quantum number m which can take on the values 0, $\pm 1, \pm 2, \pm 3, ..., \pm\infty$. There is one quantum state for each allowed value of m. The energies of the rotational states are given by $\epsilon_m = (\hbar^2/2I)m^2 \equiv \epsilon m^2$, where I is a moment of inertia. Determine the rotational energy contribution to the heat capacity C of the gas at high temperature T, where $KT \gg \epsilon$. In this limit, *sum* can be replaced by *integration* on variable m. **[12]**

BEST OF LUCK