BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
FIRST SEMESTER 2023-24
PHY F312: *Statistical Mechanics*
Midterm test (Part B)
Open Book

Total marks: 40 Time: 60 mins

Useful Integrals

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} x^n e^{-ax^2} dx = \frac{1 \cdot 3 \cdot 5..(n+1)\pi^{1/2}}{2^{n/2} a^{(n+1)/2}} \qquad n = 0, 2, 4..$$

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1. A one-dimensional random walker stands n steps from a cliff's edge. It takes **random steps**, either towards or away from the cliff, **each step independent of the past steps** . At any point, the probability of taking a step away is 2/3 , or a step toward, 1/3. What is its chances of eventually falling off the cliff?

   Work out the probability $P(1)$ where $P(n)$ represent the probability of death if you start r steps from the cliff. Start with $n = 1$ and write a general recurrence relation connecting $P(n)$ with $P(n-1)$ and $P(n+1)$

2. Consider N one-dimensional classical simple harmonic oscillators each of mass m and frequency ω. If the system is isolated, calculate the number of states with energy between E and $E + dE$ and hence find the entropy. [Use the formula for volume of a sphere in D dimensions]

3. Consider two systems -

   System $\mathcal{S}_1$: Ideal gas of N particles, each of mass m_0 and at temperature in a volume V.

   System $\mathcal{S}_2$: Ideal gas of $2N$ particles, each of mass $4m_0$ and at temperature in volume $V/2$.

   Both systems are in equilibrium with a heat bath at temperature T. For what value of T will the two systems have the same Helmholtz free energy F?

4. A zipper has N links; each link has a state in which it is closed with energy 0 and a state in which it is open with energy ϵ. We require, however, that the zipper can only unzip from the left end, and that the link number s can only open if all links to the left (1,2, ..., s — 1) are already open. Calculate the canonical partition function for a given β. [The model is a very simplified model of the unwinding of two-stranded DNA molecules]

5. N independent distinguishable particles move in one dimension between $x = 0$ and $x = L$. The system is in equilibrium at a temperature T. The single particle Hamiltonian is

   $$H = \frac{p^2}{2m} + V(x)$$

   with $V(x) = -\alpha \ln \frac{x^2}{L_0^2}$. where $\alpha > 0$ is a constant and L_0 is some characteristic length scale.

   (a) Find the partition function
   (b) Find the average $\langle x \rangle$
   (c) Find the quantity $\langle x^r \rangle$ and hence find the standard deviation of x.
   (d) Find the entropy
   (e) Find the specific heat of the gas

[(4+2) + 8 + 8 + 8 + (2 × 5)]