Mid-Term Examination I SEMESTER 2023-24 *Closed Book*

BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI

COMPUTATIONAL PHYSICS (**PHY F313**)

Date: 13-10-2023 **Max Time:** 90 min **Max Marks**: 75

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1. **[15M]** Using the Newton-Raphson method, find out the root of the equation:

$$f(x) = (x-1)^3 + 0.512.$$

Choose the initial guess as $x_0 = 5.0$ and determine the value after 10 iterations **[8]**. Calculate the error in each iteration **[4]**. Do you observe anything interesting in the values after each iteration **[3]**?

2. **[15M]** The velocity of the free-falling object is given as,

$$v(t) = \frac{g}{\alpha}(1 - e^{-\alpha t}) \qquad \text{where,} \qquad \alpha = \frac{c}{m}.$$

The velocity is in m/s while time is in s, $g = 9.8$ ms^{-2}. Find out the distance travelled by the object of mass 80 kg in the first 8 seconds of free fall.

   (a) [7M] Gaussian quadrature.

   (b) [5M] Trapezoidal method

   (c) [3M] Compare your results obtained by both the methods and comment on your results.

Discretize your integral in 10 points. The drag coefficient, c as 10 kg/s.

3. **[15M]** Find out the solution of the given differential equation

$$\frac{dx}{dt} = (1 + 2t)\sqrt{x}$$

over the interval $t = 0$ to 2 with $x(0) = 1$ using fourth order Runge Kutta method. Take $h = 1.0$.

4. **[10M]** Generate N sets of the distinct random number (between 0 and 1) by the congruential (or power series) method. Take the constants as: $a = 10, c = 100, M = 55$ and intial seed as 2.

5. **[20M]** A steady state heat balance for a rod can be represented by an equation as,

$$\frac{d^2T}{dx^2} - 0.15T = 0$$

Using the shooting method, obtain a solution for the 10-m rod with $T(0) = 240$ and $T(10) = 150$.

========== ALL THE BEST ==========

Given:

1. The Gaussian points:
   Abscissas = $\{\pm 0.9739065285, \pm 0.8650633667, \pm 0.6794095683, \pm 0.4333953941, \pm 0.148874339\}$
   Weights = $\{0.0666713443, 0.1494513492, 0.2190863625, 0.2692667193, 0.2955242247\}$