Physics Department; BITS - Pilani, Pilani

Compre-Exam. (*Part - A, CB*) ; 1^{st} **Sem' 17 - 18**

Theory of Relativity (PHY F315)

Max. Time: 3 hrs (Part - A + Part - B) **Max. Marks : 40 (Part-A) + 40 (Part - B)**

The symbols have their usual/standard meaning. Maximum time for part-A is 1.5 hrs. However, early submission is allowed, obviously. Part - B can be collected only after submission of Part - A. **Answer the parts (if any) of each question in continuation.**

Q1. Answer the following questions very briefly.

(a) Suppose a source emits a light pulse *once in a second* in its own rest frame. The source is moving away from us at speed $\frac{v}{c} = 0.8c$. How long is the interval between the pulses that we receive ?

(b) Two events (in Minkowski flat space-time) are separated by spatial distance $\Delta x = 4$ light years and temporal interval $\Delta t = 5$ years as observed from frame S. Is it possible to choose a frame S' to record the time of events in same location ? If "yes", calculate $\Delta t'$ in that frame. If "no", Justify.

(c) If A^{α} and $\frac{\partial}{\partial x^{\beta}}$ are two vectors in an arbitrary Riemannian manifold, then prove/disprove that $\frac{\partial A^{\alpha}}{\partial x^{\beta}}$ is a tensor.

(d) Justify in favour/against the statement that the Christoffel symbols, $\Gamma^{\mu}_{\alpha\beta}$ are the components of a tensor. [4 × 3]

Q2. Consider S^{ij} and $M^{i}{}_{j}$ are two symmetric tensors and A^{ij} is an anti-symmetric tensor with respect to their indices in a N-dimensional Riemannian Manifold in coordinates, $\{x^i; i = 1, ...N\}$.

(a) Assume $M^{i}{}_{j}$ transforms into $\bar{M}^{\alpha}{}_{\beta}$ under coordinate transformation, $\{x^i\}$ to $\{\bar{x}^{\alpha}\}$. Does $\bar{M}^{\alpha}{}_{\beta}$ must preserve the symmetry properties under the above coordinate transformation ? Justify your claim.

(b) Calculate the quantity, $A^{ij}\ S_{ij}$.

(c) Write the total number of independents components of S^{ij} and A^{ij} in N-dimensions.

(d) Give an example of S^{ij} and A^{ij} types of tensor from STOR or GTOR. [4 × 1]

Q3. Consider the reaction $p+p \rightarrow p+p+p+\bar{p}$, where an incoming proton hits another proton at rest and producing an antiproton ($\bar{p}$) & three protons (rest mass of proton and antiproton are same, say m_p). Determine the threshold energy E_{th} (in terms of proton mass) of the colliding proton to produce the above four particles. [Note, the threshold energy is by definition the minimum lab energy required by the incoming particle such that the momentum of each of the final particles is zero relative to CM frame.] **[10]**

Q4. Consider the spatial part of Friedmann-Robertson-Walker metric, $dS^2 = \frac{dr^2}{1-(r/a)^2} + r^2(d\theta^2 + sin^2\theta\ d\phi^2)$; "$a$" is a constant describes curvature of the space. Calculate the volume V_3 inside $r = R$ and express the result in the limit $\frac{R}{a} << 1$. [Note, $sin^{-1}x = x + \frac{x^3}{6} + ...$] **[8]**

Q5. Consider the 2-d geometry, $dS^2 = \frac{a^4}{r^4}(dr^2 + r^2\ d\phi^2)$; "a" is a constant. Is this geometry flat or curved ? Justify, whatever way you like. **[6]**

BEST WISHES

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Compre-Exam. (*Part - B, OB*) ; 1^{st} Sem' 17 - 18

Theory of Relativity (PHY F315)

Max. Time: 3 hrs (Part - A + Part - B) **Max. Marks : 40 (CB) + 40 (OB)**

The symbols have their usual/standard meaning. Text books/ref. books (Resnick, Hartle, Schutz, Weinberg), lecture notes (hand written/photocopy) are allowed.

Q1. A high-speed (relativistic) hypothetical train of proper length l_0 travels at constant velocity v relative to the ground. The front end F' of the train passes point F on the ground at $t = t' = 0$, when a light signal is sent from F' to the tail end T' of the train (see the figure).

(a) Find the times t_1 and t'_1 when the signal reaches T' as measured in the two frames.

(b) Find the times t_2 and t'_2 when the point T' passes point F as measured in the two frames. [4 + 4]

Q2. A "K" meson (rest mass, m_k) decays into two π mesons (rest mass, m_π). In the frame where one of the π mesons is at rest, find (in terms of rest mass of the particles)

(a) the energy of the other π meson and

(b) the energy of the original K meson. [5 + 3]

Q3. Assume a 2-d manifold defined by the line element in (t, x) coordinates, $ds^2 \equiv d\tau^2 = \frac{1}{t^2}(dt^2 - dx^2)$ (pls, don't worry on dimension of each term).

(a) Determine all Christoffel symbols, $\Gamma^{\alpha}_{\mu\nu}$, where $\alpha, \mu, \nu \equiv t, x$.

(b) Write down two geodesic equations (2nd order diff. eqns in t and x wrt τ) of a test particle in this manifold.

(c) Now consider the *parallel transport* of a vector $A^\mu (\mu \equiv t, x)$ from a point (t_1, x_1) to (t_2, x_2) along an arbitrary path. Find $\partial_t A^t$ and $\partial_x A^t$ in terms of A^t, A^x and t. [8 + 4 + 4]

Q4. Consider 2-d Euclidean plane in two set of coordinates, namely, Cartesian (x,y) and polar (r, θ) with corresponding metric $g_{\mu\nu}$. Where, $\mu, \nu \equiv x, y$ or r, θ depending on the set of coordinates used. Using the definition of covariant differentiation of a covariant tensor, $A_{\mu\nu;\alpha} = A_{\mu\nu,\alpha} - A_{\beta\nu}\Gamma^{\beta}_{\mu\alpha} - A_{\mu\beta}\Gamma^{\beta}_{\nu\alpha}$, determine $g_{xx\ ;x}$ and $g_{\theta\theta\ ;r}$. (Mind the comma and semi-colon, please.) [2 + 6]

BEST WISHES