Physics Department; BITS - Pilani, Pilani

Mid Term Exam (CB) ; 1^{st} Sem' 17 - 18

Theory of Relativity (PHY F315)

Max. Time: 1.5 hrs **Max. Marks : 60**

Note : Wherever applicable, the frame-S' can be considered moving with speed v relative to S along $x - x'$ axis.

Q1. Determine the following scalar quantities. The result should be expressed in terms of speed of light c, rest mass m_0, proper charge density ρ_0 etc. wherever applicable. U^μ, P^μ, a^μ and j^μ are four velocity, four momentum, four acceleration and four current, respectively. **(a)** $P^\mu U_\mu$ **(b)** $a^\mu U_\mu$ **(c)** $j^\mu j_\mu$. [3 × 3]

Q2. For the following statements, write TRUE/FALSE with self convincing justification (in one/two sentences).

(a) In any relativistic collision in a force free environment, total rest mass of the particles always conserved.

(b) The interval "$c^2(\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$" is invariant under Galilean transformation.

(c) On a Minkowski spacetime diagram, the event B lies on the light cone of an event A. These two events are separated by *timelike* interval.

(d) The *timelike* interval can be made *spacelike* by suitable Lorentz transformation.

(e) The quantity, $(|\vec{E}|^2 + |\vec{B}|^2)$ is invariant under Lorentz transformation (*clue : $F^{\mu\nu}F_{\mu\nu}$ is invariant*). [5 × 2]

Q3. Answer the following question very briefly. (3/4 steps seem to be sufficient).

(a) Justify, whether the operator $\frac{\partial}{\partial x_\mu}$ is covariant or contravariant.

(b) A charge particle (charge q and rest mass m_0) moving with instantaneous velocity $\vec{u}$ in an electromagnetic fields, $\vec{E} = 0, \vec{B} \neq 0$. Determine the 0-component of the four force f^μ of the particle.

(c) Write all elements of the matrix formed by the field strength tensor $F^{\mu\nu}$.

(d) Using Maxwell's equations, $\partial_\alpha F^{\alpha\beta} = \mu_0 j^\beta$, determine $\partial_\beta j^\beta$. (*Clue : $F^{\alpha\beta}$ is anti-symmetric.*) [4 × 4]

Q4. The rapidity variable, η of a particle (energy E and momentum $\vec{p}$ in frame S) is defined as, $\eta = ln\ (\frac{E\ +\ p_x\ c}{E\ -\ p_x\ c})$. In S', the rapidity variable η' can be written as, $\eta' = \eta + \eta(v)$. Find $\eta(v)$. [**8**]

Q5. Consider a linear collision of two identical particles (rest mass, m_0) approaching each other with energy E each. Find therelative energy E' (relativistically) of second particle wrt first. Note, E' should be expressed in terms of E and m_0. [*Clue : The quantity, $s = c^2(p_1 + p_2)^2$ is same for both cases.*] [**8**]

Q6. Consider a light source fixed at origin in frame S emitting light of frequency ν with propagation wave vector $\vec{k} = (k_x, k_y, k_z) = |\vec{k}|(cos\theta,\ sin\theta,\ 0)$. Where, θ is the angle between x-axis and the vector $\vec{k}$. Assume ν' and θ' are the corresponding frequency and angle in frame S'. Determine ν' in terms of ν and θ. [*Note, the electromagnetic four wave vector, $k^\mu = (k^0, k_x, k_y, k_z)$, where $k^0 = |\vec{k}| = \frac{2\pi\nu}{c}$*] [**9**]

BEST WISHES