

Birla Institute of Technology & Science, Pilani
K. K. Birla Goa Campus
First Semester 2022-2023

4 November 2022 Theory of Relativity (PHY F315) Mid-Semester Examination (Closed book)
 Time: 90 min Max. Marks: 60 Weightage 30%

1. Prove that the quantity

$$x^2 + y^2 + z^2 - c^2 t^2$$

is a Lorentz invariant. (5)

2. Show that two events which are simultaneous in one frame need not be simultaneous in another frame. (5)

3. In an inertial frame S , an event is observed to take place at a point A on the x -axis and 10^{-6} s later another event takes place at a point B , 900 m from A . Find the magnitude and direction of the velocity of S' with respect to S in which these two events appear simultaneous. (5)

4. Show that a photon moving at speed c (the speed of light in vacuum) will have the same speed in all frames of reference. (5)

5. Find the speed of a proton whose kinetic energy is equal to its rest mass energy. What about an electron? (5)

6. A body at rest, explodes into two bodies of rest mass 1 kg each that move apart at a speed of $0.6c$ with respect to the original body. What is the rest mass of the original body? (5)

7. (a) The *rapidity* ϕ is defined as $\tanh \phi \equiv v/c \equiv \beta$. If $\cosh \phi = a\gamma$ and $\sinh \phi = b\gamma$, find a and b .

(b) Use the previous result to rewrite the Lorentz transformation equations in terms of the rapidity.

(c) Let observer O move in the positive x' -direction of observer O' with speed β . Observer O' , in turn, moves in the positive x'' -direction of observer O'' with speed β' . Find a relation between the rapidity ϕ'' of O'' relative to O and the rapidity ϕ' of O'' relative to O'

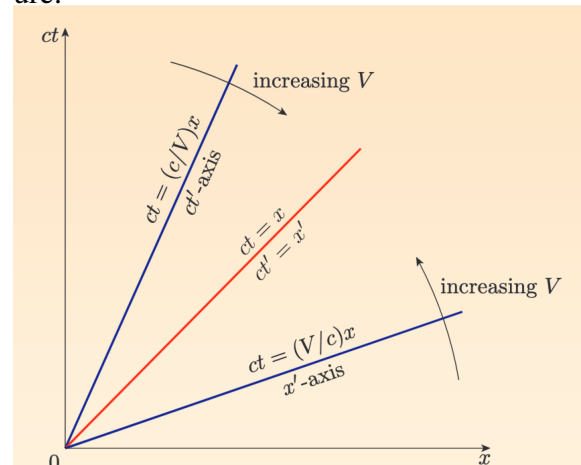
and ϕ the relative rapidity of O' relative to O . Note: $\tanh(A + B) = \frac{\tanh A + \tanh B}{1 + \tanh A \tanh B}$ (5+5+5=15)

8. Spacetime diagrams: Given two inertial frames, S and S' , in standard configuration, it is instructive to plot the ct' - and x' -axes of frame S' on the spacetime diagram for frame S . The x' -axis of frame S' is defined by the set of events for which $ct' = 0$, and the ct' -axis is defined by the set of events for which $x' = 0$. The coordinates of these events in S are related to their coordinates in S' by the usual Lorentz transformations:

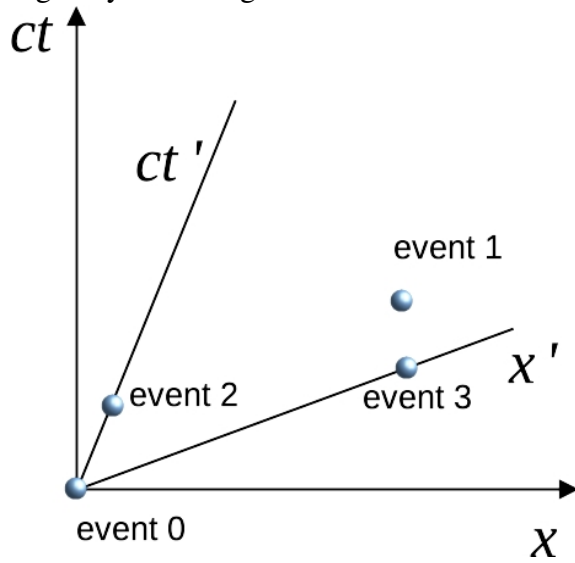
$$ct' = \gamma(V)(ct - Vx/c),$$

$$x' = \gamma(V)(x - Vt).$$

Setting $ct' = 0$ in the first of these equations gives $0 = \gamma(V)(ct - Vx/c)$. This shows that in the spacetime diagram for frame S , the x' -axis of frame S' is represented by the line $ct = (V/c)x$, a straight line through the origin with gradient V/c . Similarly, setting $x' = 0$ in the second transformation equation gives $0 = \gamma(V)(x - Vt)$, showing that the ct' -axis of frame S' is represented by the line $ct = (c/V)x$, a straight line through the origin with gradient c/V in the spacetime diagram of S . These lines are shown in the below figure.



- (a) Certain set of events, called events 0, 1, 2, 3 occur in spacetime, as shown in the below figure. Order them chronologically according to both S and S'.



- (b) Although observers might disagree about the order of some events, they will not disagree about the order of any two events that might be linked by a light signal or any other signal which travels at less than speed of light. Such events are said to be *causally related*: cause precedes effect. In the adjacent figure, which events are causally related?
- (c) For a set of two causally disconnected events in the adjacent figure, draw the spacetime diagram for a frame S'' in the frame S, in which the chronological order of these two events is reversed. (5+5+5=15)

***** That's all Folks! *****