Birla Institute of Technology & Science Pilani, K. K. Birla Goa Campus
Second Semester 2022-2023

16 May 2023 Theory of Relativity (PHY F315) Comprehensive Examination (Closed book)
Duration 3 Hrs. Max. Marks: 80 Weight 40%

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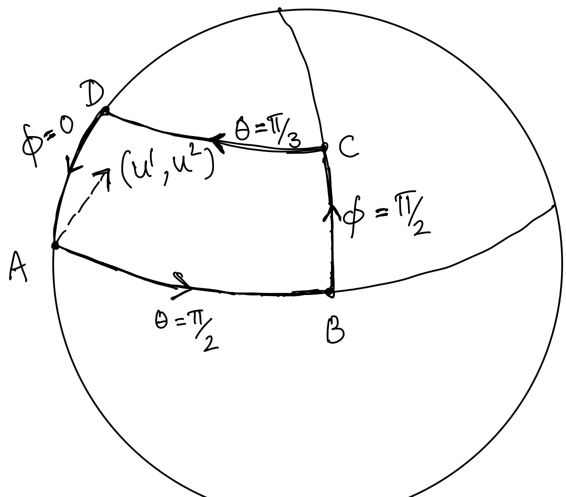
1. A rocket moves with a velocity $v = 0.8c$. As soon as it reaches a distance $d = 6.66 \times 10^8$ km from Earth, a radio signal is emitted from the Earth station to the rocket. How much time does the signal need to reach the rocket
   (a) according to a clock on the Earth station,
   (b) according to a clock on the rocket? **(5+5=10)**

2. Two electrons approach each other, each with energy γmc^2 in the laboratory frame. Compute the energy of one electron as seen in the rest frame of the other. Express your answer in terms of γ and mc^2. **(10)**

3. Consider the reaction $a + b \to c + d$. The rest masses of the particles a, b, c and d are respectively m_a, m_b, m_c and m_d. Find the threshold kinetic energy of particle a to create particle c at an angle of 90° (with respect to the incident direction) in the lab, in which the particle b is at rest before collision. Express your answer in terms of the rest masses of the particles. **(10)**

4. The inertial system S' moves relative to the inertial system S with a speed v along the common $x - x'$ axes in the positive direction. At time $t = t' = 0$ the two systems coincide. A point charge q is located at the origin of S'.
   (a) Determine the four-potentials A'^{μ} and A^{μ} in S' and S, respectively.
   (b) Using A^{μ}, calculate the electromagnetic fields $\mathbf{E}$ and $\mathbf{B}$ of the point charge in S.
   (c) What is the vector equation connecting $\mathbf{E}, \mathbf{B}$ and $\mathbf{v}$? **(6+6+3=15)**

5. Consider a contravariant *unit* vector on the surface of a unit sphere with polar coordinates θ, ϕ ($\theta = 0$ being the north pole and $\theta = \pi/2$ the equator). It is parallely transported along the equator from $\phi = 0$ to $\phi = \pi/2$, then similarly transported along the meridian ($\phi = \pi/2$) from $\theta = \pi/2$ to $\theta = \pi/3$ and then along the latitude ($\theta = \pi/3$) from $\phi = \pi/2$ to $\phi = 0$. Finally, it is transported similarly along the meridian back to the starting point $\theta = \pi/2, \phi = 0$ (see figure below). Find the angle between the initial and final vectors. Note: If $x^1 \equiv \theta$ and $x^2 \equiv \phi$, $g_{11} = 1, g_{22} = \sin^2\theta$ and the non-zero connection coefficients/Christoffel symbols are $\Gamma^1_{22} = -\sin\theta\cos\theta$ and $\Gamma^2_{12} = \cot\theta$. Note: The scalar product of two vectors $\mathbf{A}$ and $\mathbf{B}$ is $\mathbf{A} \cdot \mathbf{B} \equiv \sum_i A^i B_i = \sum_{i,j} g_{ij} A^i B^j$. **(15)**



6. Consider a torus in a two-dimensional Euclidean space described by the spherical coordinate system (θ, ϕ). The line element of the torus is then given by

$$\mathrm{d}s^2 = (b + a\sin\phi)^2\, \mathrm{d}\theta^2 + a^2\, \mathrm{d}\phi^2,$$

   where b and a denote the torus radius and the radius of its section, respectively.
   (a) Find all the components of the metric tensor g_{ij} and its duel, g^{ij}.
   (b) Compute all the non-zero Christoffel symbols/connection coefficients.
   (c) Find the only independent non-zero component of the (Riemann) curvature tensor, proving that the space is curved. **(4+8+8=20)**