Birla Institute of Technology & Science, Pilani
K. K. Birla Goa Campus
Second Semester 2022-2023

16 March 2023 Theory of Relativity (PHY F315) Mid-Semester Examination (Closed book)
Duration 90 min Max. Marks: 60 Weight 30%

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1. A Deer is running at a speed (3/4)c. A hunter, pursuing it in a car which can travel at (1/2)c, shoots a bullet from his car. The muzzle velocity of the bullet (relative to the gun) is (1/3)c. Does the hunter manage to hit the deer (a) according to Galileo, (b) according to Einstein? **(5+5=10)**

2. Consider a collection of particles, all moving in the x direction, with energies $E_1, E_2, E_3, \ldots$ and momenta $p_1, p_2, p_3, \ldots$ Find the velocity of the center of momentum frame, in which the total momentum is zero. **(10)**

3. The four-dimensional gradient operator $[\partial/\partial x^\mu]$ functions like a covariant 4-vector. In fact, it is often written as $[\partial_\mu]$, for short. The corresponding contravariant gradient would be $[\partial^\mu \equiv \partial/\partial x_\mu]$. Prove that $[\partial^\mu \phi]$ is a (contravariant) 4-vector, if ϕ is a scalar function, by working out its transformation law, using the chain rule. **(10)**

4. Inertial system S' moves at constant velocity $\mathbf{v} = v(\cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}})$ with respect to frame S. Their axes are parallel to one another, and their origins coincide at $t = t' = 0$, as usual. Find the Lorentz transformation matrix L. Hint: What is the transformation rule for coordinates, which are related via a rotation about the z-axis? **(10)**

5. A particle of rest mass m_1 and velocity $\overrightarrow{v_1}$ collides with a stationary particle of rest mass m_2 and is absorbed by it. Find the rest mass m and the velocity $\vec{v}$ of the resultant compound system in terms of m_1, m_2 and $v_1 (= |\overrightarrow{v_1}|)$. **(10)**

6. If two frames have 4-velocities, U_1 and U_2, prove that

$$U_1 \cdot U_2 = \gamma,$$

the Lorentz transformation factor, between the two frames. **(10)**
