Birla Institute of Technology and Science, Pilani
Mid-Semester Examinatiom
Theory of Relativity (PHY F315)

Date : 02.11.2022 **Maximum Marks : 90** **Time : 90 Minutes**

Q1) Consider two frames, S & S', with axes parallel. Frame S' is moving with velocity v with respect to S along the latter's x-axis. A particle describes a uniform circular motion of radius R in frame S' around its origin and on the $x'-y'$ plane. The equations of motion of the particle, as observed in the S' frame are thus

$$x' = R\cos\omega t' \;;\; y' = R\sin\omega t'$$

a) Apply Lorentz transformations to obtain the equations of motion of the particle in the S frame.

b) In the S frame, the centre of motion of the particle (i.e., the origin of S' frame) moves with velocity v along the x-axis. Denoting the coordinates of the particle, with respect to the moving centre of motion by $\bar{x}$ & $\bar{y}$, write down the equations of motion of part a in terms of $\bar{x}$ & $\bar{y}$.

[Note that $\bar{x}$ & $\bar{y}$ are more relevant coordinates for the S frame observer than x & y]

c) Show that $\bar{x}$ & $\bar{y}$ are also periodic in time t in the S frame, and find the time period T.

[Show that the equations satisfied by $\bar{x}$ & $\bar{y}$ remain the same at $t+T$ as at t, for some appropriate T which you have to find. Note that the coordinates $\bar{x}$ & $\bar{y}$ cannot be explicitly expressed as functions of time]

d) Since the particle going round in a circle periodically can be the model of a clock, how would you interpret the relationship between the time periods in frames S & S'?

e) By eliminating time t between $\bar{x}$ & $\bar{y}$, show that the particle, as seen from the S frame, describes an ellipse around the moving centre (origin of S'). Explain the size of the two axes of this ellipse (major and minor) as a relativistic effect. (5+7+7+4+7)

Q2) Consider a particle moving along the x-axis of a frame S as given below.

$$x(t)=\sqrt{b^2+t^2} \qquad t \geq 0$$

Note that we are using relativistic units, so that the constant 'b' has the dimension of time.

a) Find the velocity of the particle as a function of time in frame S and show that it never exceeds the speed of light. (3)

b) Find the proper time τ as a function of time t in frame S, assuming that both τ & t are zero when the particle starts moving from $x=b$. (10)

c) What time will the particle's clock read when the S frames clock reads 10 years? Take the value of b to be 1 year in relativistic units. (4)

d) Draw the particle's world line on the space-time diagram on the x-t plane for $t \geq 0$. (5)

e) Another particle moves in exactly the same manner but along the negative x-axis. Its position as a function of time is thus

$$x(t) = -\sqrt{b^2 + t^2} \qquad t \geq 0$$

Mark points on the world lines of each, beyond which the particles cannot communicate with each other even at the speed of light, i.e., they cannot exchange photons with each other. (8)

Q3a) Show that the wave equation for waves propagating with the speed of light, given as

$$\frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi = 0 \text{ or } \frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi = 0 \text{ (in relativistic units)}$$

Can be written down as

$$\eta^{\mu\nu}\frac{\partial^2 \phi}{\partial x^\mu \partial x^\nu} = 0 \qquad (4)$$

[Note that $\phi(x^\mu)$, the wave amplitude, is a scalar function]

b) Prove that the above equation (the one written using the metric tensor) remains invariant under Lorentz transformation, i.e., it is a scalar equation. Hence conclude that the speed of a wave, propagating with the speed of light, is the same in all inertial frames. (8)

[You may use the transformation rule for $\eta^{\mu\nu}$ as a second rank contavariant tensor]

Q4) A mass M at rest splits into two smaller masses, $\frac{M}{2}$ & $\frac{M}{4}$ respectively.

a) Find the total energy that each will carry after the split. (12)

b) Find the velocities of the two masses. (6)

[It will help to begin with the 4-momentum conservation equation]

Or

a) Prove that it is always possible to find a Lorentz frame in which the total 3-momentum of a system of particles is zero. (12)

b) If two masses, m each, are moving with velocities $\frac{3}{5}$ & $\frac{4}{5}$ along the x-axis and y-axis of the lab frame respectively, then find the velocity vector of the CM frame. (6)

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