Open Book

Q1) Consider a charged particle of charge q (assume it to be positive) and rest mass m, that is fired with a velocity v_0 against a uniform electric field of magnitude E_0. Take the motion to be along the positive x-axis and the field to be along the negative x-axis. The relativistic equation of motion of the particle is thus

$$\frac{d(m\gamma v)}{dt} = -qE_0$$

a) Show that when the equation of motion is written down in terms of the independent variable x (in place of time t) it becomes

$$v\gamma^3 \frac{dv}{dx} = -\frac{qE_0}{m}$$

b) Integrate the above equation to find the velocity of the particle as a function of x, and hence find the distance travelled by the particle before coming to rest.

c) Find the stopping potential (potential difference that the particle has to travel against before coming to rest) and show that it reduces to the classical value in the classical limit.

[Potential difference for a constant field is $\Delta\phi = E_0\Delta x$] (8 + 12 + 5)

Q2) Consider a spherical satellite of radius a going around the earth at a radius R (distance between the centre of the earth and that of the satellite).

a) From the distribution of the tidal gravity inside the satellite, calculate the tidal gravitational potential difference between the centre of the satellite and the farthest point on the satellite from the centre of the earth.

b) Assuming that effects of tidal gravitational fields are similar to those of real gravitational fields, calculate the frequency shift of light emitted at the farthest point as it reaches the centre of the satellite. Take $a = 10\,\text{m}$, $R = 7\times10^6\,\text{m}$ and that at this value of R, $g = 8.45\,\text{m/s}^2$. Is the shift red shift or blue shift?

c) Will there be any frequency shift when light emitted from the farthest point is received at the nearest point (farthest and nearest defined w.r.t. the centre of the earth). Answer it, if you can, without doing any detailed calculations. (12 + 8 + 5)

Q3) The electromagnetic stress tensor is defined as ($F^{\mu\nu}$ is the electromagnetic field tensor)

$$T^{\mu\nu} = \varepsilon_0\left(\frac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta} - F^{\mu\alpha}F^{\nu}{}_{\alpha}\right)$$

a) Show that the RHS of the above equation is indeed a tensor of rank $(2,0)$. Be brief in your explanation and cite only the rules of tensor algebra you use in proving your points. (5)

b) Calculate the component T^{00} and state the physical meaning of it. (8)

c) Prove that $T^{\mu\nu}$ is traceless, i.e., $T^{\mu}{}_{\mu} = 0$. (7)

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Birla Institute of Technology and Science, Pilani
Comprehensive Examination
Theory of Relativity (PHY F315)

Date : 22.12.2022 **Maximum Marks : 120** **Time : 3 hours**

Closed-Book

Q1) As seen from the earth, two supernovae, separated by 10 light years, explode with a time gap of 2 years. For simplicity, orient the axes of the earth's frame so that the two supernovae are on the x-axis and the supernova that exploded first has a smaller x-coordinate.

a) Calculate the space-time interval between the two events (the two explosions) in meters. Are the two events space, time or light like separated? (4)

b) Find the velocity of an observer, moving w.r.t to the earth along the line joining the two supernovae, so that he finds that the supernova with smaller x-coordinate, exploded one year later than the other. (8)

Q2) Consider a long street (x-axis) on which there are lamp posts at regular intervals, ℓ. A car is speeding away at a speed $v = 1/\sqrt{3}$ along this street . The lamps on the lamp posts are switched on simultaneously in the ground frame at t = 0.

a) Draw the space and time axes of the two frames, the ground frame and the car's frame. (4)

b) Mark on the time axis of the car's frame, the time instants when the lamps are switched on according to the driver of the car. What is the time gap Δt between the switching-on of two consecutive lamps as seen by the driver of the car by taking ℓ to be i) 100 m ii) one light year. (10)

Q3) Generate the following raising and lowering of indices with the help of the metric tensor.

$$\text{i) } T_{\lambda}^{\ \mu\nu} \rightarrow T^{\lambda\ \nu}_{\ \mu} \quad \text{ii) } T_{\lambda}^{\ \mu\nu} \rightarrow T_{\lambda\mu\nu}$$ (6)

b) Prove that if $F^{\mu\nu}$ is a symmetric or anti-symmetric tensor then so is $F_{\mu\nu}$. (6)

Q4a) Derive the expression for the metric tensor $g_{\mu\nu}$ in curvilinear coordinates in terms $\eta_{\mu\nu}$ and the local Cartesian coordinates ξ^{μ}.

b) Using the expression for $g_{\mu\nu}$ in part (a), show that

$$g'_{\mu\nu} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}$$ (6 + 6)

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